

A compendium of logarithmic corrections in AdS/CFT

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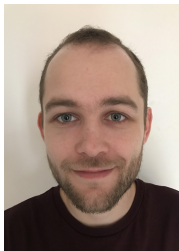
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Motivation

Supersymmetric localization allows for the exact calculation of physical observables in supersymmetric QFTs.

Apply this tool to SCFTs with holographic duals in string and M-theory.

$$Z_{\text{CFT}}[J] = Z_{\text{string/M}}[\phi].$$

Focus on subleading terms in the large N expansion of $Z_{\text{CFT}}[J]$ to learn about quantum corrections to the supergravity approximation of $Z_{\text{string/M}}[\phi]$.

A new handle on AdS vacua of string and M-theory with non-trivial fluxes.

Learn about quantum corrections to black hole thermodynamics.

Goal: Describe recent progress on these topics for 3d SCFTs with AdS_4 duals in M-theory focusing on the $\log N$ terms in the large N expansion.

Plan

- Motivation ✓
- Supersymmetric partition functions
- Log corrections from supergravity
- The unbearable lightness of the KK scale
- Breaking supersymmetry: black holes and thermal observables
- Outlook

Supersymmetric partition functions

[Aharony,Bergman,Jafferis,Maldacena]; [Kapustin,Willet,Yaakov]; [Drukker,Mariño,Putrov]; [Mariño,Putrov];
[Fuji,Hirano,Moriyama]; [Herzog,Klebanov,Pufu,Tesileanu]; [Benini,Zaffaroni]; [Closset, Kim];
[Benini,Hristov,Zaffaroni]; [Liu,Pando Zayas,Rathee,Zhao]; [NPB,Hong,Reys]; [Nosaka]; [Hatsuda]; [Hristov];
[Chester,Kalloor,Sharon]; [Bhattacharya²,Minwalla,Raju]; [Kim]; [Choi,Hwang,Kim]; [Choi,Hwang];
[Nian,Pando Zayas]; [NPB,Choi,Hong,Reys]; [NPB,De Smet,Hong,Reys,Zhang]; [Kubo-Nosaka-Pang]

ABJM and holography

The ABJM theory: $U(N)_k \times U(N)_{-k}$ 3d CS-matter theory with two pairs of bi-fundamental chirals $(A_{1,2}, B_{1,2})$ and superpotential

$$\mathcal{W} = \text{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1).$$

For $k > 2$ it has $\mathcal{N} = 6$ supersymmetry and $SU(4)_R \times U(1)_b$ global symmetry. Describes N M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$.

- In the limit of fixed k and large N , the ABJM theory is dual to the M-theory background $\text{AdS}_4 \times S^7/\mathbb{Z}_k$

$$(L/\ell_{\text{P}})^6 \sim k N.$$

- At large k and fixed 't Hooft coupling $\lambda = N/k$ the theory is dual to type IIA string theory on $\text{AdS}_4 \times \mathbb{CP}^3$

$$k g_{\text{st}} = L/\ell_{\text{s}} \sim \lambda^{1/4}.$$

Perturbative type IIA string theory at large k and small g_{st} , i.e. fixed λ and large N .

ABJM on S^3

The path integral on S^3 can be computed by supersymmetric localization and reduces to a matrix model

$$Z(N, k) = \frac{1}{N!^2} \int \frac{d^N \mu}{(2\pi)^N} \frac{d^N \nu}{(2\pi)^N} \exp \left[\frac{ik}{4\pi} \sum_{i=1}^N (\mu_i^2 - \nu_i^2) \right] \frac{\prod_{i < j} \left[2 \sinh\left(\frac{\mu_i - \mu_j}{2}\right) \right]^2 \left[2 \sinh\left(\frac{\nu_i - \nu_j}{2}\right) \right]^2}{\prod_{i,j} \left[2 \cosh\left(\frac{\mu_i - \nu_j}{2}\right) \right]^2}$$

Different methods have been used to study $Z(N, k)$ at large N

- Map to CS theory on S^3/\mathbb{Z}_2 (or topological strings on $\mathbb{P}^1 \times \mathbb{P}^1$) and solve with large N techniques. Applies at large N , fixed N/k .
- Study the large N limit at fixed k numerically.
- Map the problem to a free Fermi gas on the real line with non-standard kinetic term. Valid at large N and finite k .

ABJM on S^3 - An Airy Tale

The path integral on a squashed S^3 with real mass deformation can be computed by supersymmetric localization and reduces to a matrix model.

$$Z_{S^3}(N, k, m_a, b) \stackrel{!}{=} e^{\mathcal{A}(k, \Delta, b)} C^{-\frac{1}{3}} \text{Ai}[C^{-\frac{1}{3}}(N - B)] + \mathcal{O}(e^{-\sqrt{N}}),$$

with fixed k , large N and

$$C = \frac{2}{\pi^2 k} \frac{(b + b^{-1})^{-4}}{\prod_{a=1}^4 \Delta_a}, \quad B = \frac{k}{24} - \frac{1}{12k} \sum_{a=1}^4 \frac{1}{\Delta_a} + \frac{1 - \frac{1}{4} \sum_a \Delta_a^2}{3k(b + b^{-1})^2 \prod_{a=1}^4 \Delta_a},$$

and

$$\begin{aligned} \Delta_1 &= \frac{1}{2} - i \frac{m_1 + m_2 + m_3}{b + b^{-1}}, & \Delta_2 &= \frac{1}{2} - i \frac{m_1 - m_2 - m_3}{b + b^{-1}}, \\ \Delta_3 &= \frac{1}{2} + i \frac{m_1 + m_2 - m_3}{b + b^{-1}}, & \Delta_4 &= \frac{1}{2} + i \frac{m_1 - m_2 + m_3}{b + b^{-1}}. \end{aligned}$$

The large N expansion takes the explicit form

$$-\log Z_{S^3} = \frac{2}{3\sqrt{C}} N^{\frac{3}{2}} - \frac{B}{\sqrt{C}} N^{\frac{1}{2}} + \frac{1}{4} \log N - \mathcal{A} + \frac{1}{4} \log \frac{32}{k} + \mathcal{O}(N^{-\frac{1}{2}}).$$

The topologically twisted index

The topologically twisted index (TTI) is the partition function of 3d $\mathcal{N} = 2$ SCFTs on $S^1 \times \Sigma_g$. Supersymmetry is preserved by a topological twist on Σ_g .

Using supersymmetric localization the path integral can be reduced to a matrix integral and computed at large N and fixed k . The free energy $F_{S^1 \times \Sigma_g} = -\log Z_{S^1 \times \Sigma_g}$, takes the simple form:

$$F_{S^1 \times \Sigma_g} = \frac{\pi \sqrt{2k \Delta_1 \Delta_2 \Delta_3 \Delta_4}}{3} \sum_{a=1}^4 \frac{n_a}{\Delta_a} \left(\hat{N}_\Delta^{\frac{3}{2}} - \frac{c_a}{k} \hat{N}_\Delta^{\frac{1}{2}} \right) + \frac{1-g}{2} \log \hat{N}_\Delta - \hat{f}_0(k, \Delta, \mathbf{n}) + \mathcal{O}(e^{-\sqrt{N}}),$$

where $\sum_{a=1}^4 \Delta_a = 2$, $\sum_{a=1}^4 n_a = 2(1-g)$, and

$$\hat{N}_\Delta \equiv N - \frac{k}{24} + \frac{1}{12k} \sum_{a=1}^4 \frac{1}{\Delta_a}, \quad c_a = \frac{\prod_{b \neq a} (\Delta_a + \Delta_b)}{8 \Delta_1 \Delta_2 \Delta_3 \Delta_4} \sum_{b \neq a} \Delta_b.$$

The holographic dual is given by (Euclidean) supersymmetric static Reissner-Nordström BHs in AdS_4 . The TTI computes the entropy of these BHs.

The superconformal index

The superconformal index (SCI), or $S^1 \times_\omega S^2$ partition function, counts $\frac{1}{16}$ -BPS operators in 3d $\mathcal{N} = 2$ SCFTs.

It is useful to consider the Cardy-like limit $\omega \rightarrow 0$. The SCI can then be analyzed with similar tools as the TTI.

For the ABJM theory at fixed k and large N we find the following ω^{-1} and ω^0 results (for $\Delta_a = 1/2$)

$$\begin{aligned} \log Z_{S^1 \times_\omega S^2}(N, k, \omega) &= -\frac{\pi\sqrt{2k}}{3} \left[\left(\frac{1}{2\omega} + 1 \right) \left(N - \frac{k}{24} + \frac{2}{3k} \right)^{\frac{3}{2}} - \frac{3}{k} \left(N - \frac{k}{24} + \frac{2}{3k} \right)^{\frac{1}{2}} \right] \\ &\quad - \frac{2}{\omega} \hat{g}_0(k) - \frac{1}{2} \log \left(N - \frac{k}{24} + \frac{2}{3k} \right) + \hat{f}_0(k) + \mathcal{O}(e^{-\sqrt{N}}) + \mathcal{O}(\omega). \end{aligned}$$

This index captures the entropy of supersymmetric AdS₄ Kerr-Newman black holes.

log N results in 3d $\mathcal{N} = 2$ susy QFT

Theory	\mathcal{M}_3	log coefficient C	Ref.	10/11d bulk
M2-brane theories (class I)				
$(S^7/\mathbb{Z}_k)_{\text{free}} \text{ (†)}$	S_b^3	$-\frac{1}{4}$	[25, 26, 44, 53]	✓ [16] (s.c.)
	$S^1 \times \Sigma_{\mathfrak{g}}$	$-\frac{1}{2}(1 - \mathfrak{g})$	[18, 44]	✓ [17] (s.c.)
	$S^1 \times_{\omega} S^2$	$-\frac{1}{2}$	[45]	✗
$(S^7/\mathbb{Z}_{N_f})_{\text{f.p.}} \text{ (†)}$	S_b^3	$-\frac{1}{4}$	[40, 42, 46, 54]	✗
	$S^1 \times \Sigma_{\mathfrak{g}}$	$-\frac{1}{2}(1 - \mathfrak{g})$	[46]	
	$S^1 \times_{\omega} S^2$	$-\frac{1}{2}$	[45]	
N^{010}/\mathbb{Z}_k	$S_{b=1}^3$	$-\frac{1}{4}$	[26, 46]	✗
	$S^1 \times \Sigma_{\mathfrak{g}}$	$-\frac{1}{2}(1 - \mathfrak{g})$	[46]	✓ [20]
	$S^1 \times_{\omega} S^2$	$-\frac{1}{2}$	[55]	✗
V^{52}/\mathbb{Z}_k	$S^1 \times \Sigma_{\mathfrak{g}}$	$-\frac{1}{2}(1 - \mathfrak{g})$	[46]	✓ [20]
	$S^1 \times_{\omega} S^2$	$-\frac{1}{2}$	[55]	✗
Q^{111}/\mathbb{Z}_k	$S^1 \times \Sigma_{\mathfrak{g}}$	$-\frac{1}{2}(1 - \mathfrak{g})$	[46]	✓ [20]
	$S^1 \times_{\omega} S^2$	$-\frac{1}{2}$	[55]	
M5-brane theories (class II)				
A_{N-1}	S_b^3	$-\frac{1}{2}$	[56, 57]	✗
	$S^1 \times \Sigma_{\mathfrak{g}>1}$	$(b_1(\mathcal{H}_3) - 1)(1 - \mathfrak{g})$	[19, 58]	
	$S^1 \times_{\omega} S^2$	$b_1(\mathcal{H}_3) - 1$	[58]	✓ [58]
D_N	S_b^3	0	[57]	✗
	$S^1 \times \Sigma_{\mathfrak{g}>1}$			
	$S^1 \times_{\omega} S^2$			
IIA theories (class III)				
$\mathbb{C}\mathbb{P}^3 \text{ (†)}$	S_b^3	$-\frac{1}{6}$	[44, 59]	✗
	$S^1 \times \Sigma_{\mathfrak{g}}$	$\frac{2}{3}(1 - \mathfrak{g})$	[44, 60]	
$\mathbb{C}\mathbb{P}_{\text{def}}^3$	$S_{b=1}^3$	$-\frac{1}{6}$	[61]	✗
S_{def}^3	$S_{b=1}^3$	$-\frac{2}{9}$ (fixed k)	[62]	✗
		$-\frac{1}{6}$ ('t Hooft)		
	$S^1 \times \Sigma_{\mathfrak{g}}$	$-\frac{7}{18}(1 - \mathfrak{g})$	[63]	

Log corrections from supergravity

[Camporesi,Higuchi]; [Vassilevich]; [Sen]; [Bhattacharyya,Grassi,Mariño,Sen]; [Liu,Pando Zayas,Rathee,Zhao];
[Pando Zayas,Xin]; [Hristov,Reys]; [David,Godet,Liu,Pando Zayas]; [NPB-David-Hong-Reys-Zhang]

Log corrections

There are log corrections to the (semi-classical) BH entropy (Area $\gg G_N$)

$$S_{\text{BH}} = \frac{\text{Area}}{4G_N} + s_0 \log \frac{\text{Area}}{G_N} + \dots$$

Ashoke Sen: s_0 can be computed by 1-loop contributions of all “light” fields in the BH background. Agreement with string theory UV calculations for BPS black holes. “IR window into UV physics!”

Example: 4d Schwarzschild in GR + n_s fields of spin $s = 0, \frac{1}{2}, 1, \frac{3}{2}$.

$$s_0 = \frac{1}{90} \left(2n_0 + 7n_{\frac{1}{2}} - 26n_1 - \frac{233}{2}n_{\frac{3}{2}} + 289 \right).$$

Here: Log corrections in AdS₄, i.e. $\log \frac{L^2}{G_N}$ with $L^2 \gg G_N$.

$$F_{S^3}(b, \Delta) = f_{\frac{3}{2}}(b, \Delta)N^{\frac{3}{2}} + f_{\frac{1}{2}}(b, \Delta)N^{\frac{1}{2}} + \frac{1}{4} \log N + \dots$$

$$F_{S^1 \times \Sigma_g}(\mathbf{n}, \Delta) = g_{\frac{3}{2}}(\mathbf{n}, \Delta)N^{\frac{3}{2}} + g_{\frac{1}{2}}(\mathbf{n}, \Delta)N^{\frac{1}{2}} + \frac{1-g}{2} \log N + \dots$$

$$F_{S^1 \times_{\omega} S^2}(\omega) = h_{\frac{3}{2}}(\omega)N^{\frac{3}{2}} + h_{\frac{1}{2}}(\omega)N^{\frac{1}{2}} + \frac{1}{2} \log N + \dots$$

The coefficient of $\log N$ does NOT depend on continuous parameters!

Heat kernel in 4d

Study the log term in the (Euclidean) path integral of GR+EFT in AdS₄ with cutoff scale Λ

$$-\log Z_{\text{GR+EFT}} = \frac{1}{16\pi G_{\text{N}}} S_{\text{cl}}(\overset{\circ}{\phi}) + \mathcal{C} \log L\Lambda + \dots$$

All fields ϕ with $\text{mass}_\phi < \Lambda$ contribute to \mathcal{C} . Use the heat kernel method to compute \mathcal{C} .

Input: The kinetic operator \mathcal{Q}_ϕ and the number of zero modes

$$\mathcal{C} = \sum_{\phi} \int d^4x \sqrt{g} a_4(x, \mathcal{Q}_\phi) + \mathcal{C}_{\text{ZM}}.$$

The Seeley-de Witt coefficient $a_4(x, \mathcal{Q}_\phi)$ depends on the background fields

$$16\pi^2 a_4(x, \mathcal{Q}_\phi) = a_E E_4 + c W^2 + b_1 R^2 + b_2 R F_{\mu\nu} F^{\mu\nu}.$$

Straightforward to calculate $a_4(x, \mathcal{Q}_\phi)$ for massive fields of spin ≤ 2 .

Subtlety: It is in general hard to compute \mathcal{C}_{ZM} . Rigorous results only for AdS₄ and AdS₂ \times Σ_g .

Massive scalar

The background: Any solution of 4d Einstein-Maxwell theory

$$S_{\text{EM}} = -\frac{1}{16\pi G_{\text{N}}} \int d^4x \sqrt{q} \left[R + \frac{6}{L^2} - F_{\mu\nu} F^{\mu\nu} \right].$$

The field: Scalar fields ϕ of mass m , charge q and action

$$S_\phi = \int d^4x \sqrt{g} \phi [-\mathcal{D}^\mu \mathcal{D}_\mu + m^2] \phi, \quad \mathcal{D}_\mu = \nabla_\mu - iqA_\mu.$$

The Seeley-de Witt coefficients:

$$a_E = \frac{1}{360}, \quad c = \frac{1}{120}, \quad b_1 = \frac{1}{288} [(mL)^2 + 2]^2, \quad b_2 = \frac{1}{144} (qL)^2.$$

Seeley-de Witt Coefficients

Bulk contribution to the SdW coefficient for massive fields of spin ≤ 2 .

spin	mass	a_E	c	b_1
0	$(mL)^2 = -2$	$\frac{1}{360}$	$\frac{1}{120}$	0
0	m	$\frac{1}{360}$	$\frac{1}{120}$	$\frac{1}{288} \left((mL)^2 + 2 \right)^2$
1/2	0	$-\frac{11}{720}$	$-\frac{1}{40}$	0
1/2	m	$-\frac{11}{720}$	$-\frac{1}{40}$	$\frac{1}{144} (mL)^2 \left((mL)^2 - 2 \right)$
1	0	$\frac{31}{180}$	$\frac{1}{10}$	0
1	m	$\frac{31}{180} + \frac{1}{360}$	$\frac{1}{10} + \frac{1}{120}$	$\frac{1}{288} \left(3(mL)^4 - 12(mL)^2 + 4 \right)$
3/2	$mL = 1$	$\frac{589}{720}$	$\frac{137}{120}$	0
3/2	m	$\frac{589}{720} - \frac{11}{720}$	$\frac{137}{120} - \frac{1}{40}$	$\frac{1}{72} \left((mL)^4 - 8(mL)^2 + 11 \right)$
2	0	$\frac{571}{180}$	$\frac{87}{20}$	0
2	m	$\frac{571}{180} + \frac{31}{180} + \frac{1}{360}$	$\frac{87}{20} + \frac{1}{10} + \frac{1}{120}$	$\frac{5}{288} \left((mL)^4 - 8(mL)^2 + 8 \right)$

Important: For $s = 1, \frac{3}{2}, 2$ massless fields need ghosts while massive fields need Stückelberg "friends".

Log-Bootstrap

Study 4d sugra backgrounds and impose that \mathcal{C} does not depend on continuous parameters.

- AdS-Taub-NUT ($U(1) \times U(1)$ squashed S^3)

$$ds^2 = f_1^2 dx^2 + f_2^2 dy^2 + \frac{1}{f_1^2} (d\psi + y^2 d\phi)^2 + \frac{1}{f_2^2} (d\psi + x^2 d\phi)^2,$$

$$f_1^2 = \frac{L^2(y^2 - x^2)}{(x^2 - 1)(b^4 - x^2)}, \quad f_2^2 = \frac{L^2(y^2 - x^2)}{(y^2 - 1)(y^2 - b^4)},$$

$$F = d \left[\frac{b^4 - 1}{L(x + y)} (d\psi - xy d\phi) \right].$$

- Euclidean Romans solution (Euclidean susy RN black hole in AdS_4)

$$ds^2 = \left[\left(\frac{r}{L} + \frac{\kappa L}{2r} \right)^2 - \frac{q^2}{4r^2} \right] d\tau^2 + \left[\left(\frac{r}{L} + \frac{\kappa L}{2r} \right)^2 - \frac{q^2}{4r^2} \right]^{-1} dr^2 + r^2 ds_{\Sigma_g}^2,$$

$$F = \frac{q}{r^2} d\tau \wedge dr - \kappa L \text{vol}_{\Sigma_g}.$$

This leads to the strong constraint

$$c^{\text{tot}} = b_1^{\text{tot}} = b_2^{\text{tot}} = 0$$

Top-down KK supergravity

Consider the concrete example of 11d sugra on S^7 dual to the ABJM theory at $k = 1$ and large N .

The resulting 4d $\mathcal{N} = 8$ gauged sugra is not a standard EFT, it has infinitely many fields!

Organize the KK modes into $\mathcal{N} = 8$ multiplets and compute the SdW coefficients.

Spectrum on $\text{AdS}_4 \times S^7$

spin	Dynkin label	Δ
2	$(n, 0, 0, 0)_{n \geq 0}$	$3 + \frac{n}{2}$
$3/2$	$(n, 0, 0, 1)_{n \geq 0}$ $(n-1, 0, 1, 0)_{n \geq 1}$	$\frac{5}{2} + \frac{n}{2}$ $\frac{7}{2} + \frac{n}{2}$
1	$(n, 1, 0, 0)_{n \geq 0}$ $(n-1, 0, 1, 1)_{n \geq 1}$ $(n-2, 1, 0, 0)_{n \geq 2}$	$2 + \frac{n}{2}$ $3 + \frac{n}{2}$ $4 + \frac{n}{2}$
$1/2$	$(n+1, 0, 1, 0)_{n \geq 0}$ $(n-1, 1, 1, 0)_{n \geq 1}$ $(n-2, 1, 0, 1)_{n \geq 2}$ $(n-2, 0, 0, 1)_{n \geq 2}$	$\frac{3}{2} + \frac{n}{2}$ $\frac{5}{2} + \frac{n}{2}$ $\frac{7}{2} + \frac{n}{2}$ $\frac{9}{2} + \frac{n}{2}$
0_+	$(n+2, 0, 0, 0)_{n \geq 0}$ $(n-2, 2, 0, 0)_{n \geq 2}$ $(n-2, 0, 0, 0)_{n \geq 2}$	$1 + \frac{n}{2}$ $3 + \frac{n}{2}$ $5 + \frac{n}{2}$
0_-	$(n, 0, 2, 0)_{n \geq 0}$ $(n-2, 0, 0, 2)_{n \geq 2}$	$2 + \frac{n}{2}$ $4 + \frac{n}{2}$

Massive $\mathcal{N} = 8$ supermultiplets at KK level n .

Top-down KK supergravity

At each KK level n one has $c(n) = b_1(n) = b_2(n) = 0!$

For the total a_E coefficient one finds the divergent sum

$$a_E = -\frac{1}{72} \sum_{n=0}^{\infty} (n+1)(n+2)(n+3)^2(n+4)(n+5).$$

Unclear how to regulate this sum. If we take $a_E = -1/3$ we find

$$\boxed{\mathcal{C}(\partial\mathcal{M}) = -\frac{1}{4}\chi(\mathcal{M})}.$$

Perfect agreement with all susy localization results in the ABJM theory!

Similar results for a number of other AdS_4 vacua in M-theory with explicitly known KK spectra.

The unbearable lightness of the KK scale

Assumption: The UV completion of GR+EFT in AdS_4 is holographic, i.e. there is a dual family of 3d CFTs with a suitable large N limit.

Consider such a GR+EFT with **finitely** many fields and a cutoff Λ such that $L\Lambda \sim N^\alpha$ with $\alpha > 0$ (or $L\Lambda \sim \lambda^\beta$ for a marginal coupling λ).

The free energy of the 3d CFT on a compact Euclidean manifold takes the form

$$\log Z_{\text{CFT}} = F_0 + \mathcal{C}_{\log} \log N ,$$

where F_0 contains all positive powers of N .

If \mathcal{C}_{\log} does **not** depend on continuous parameters (mass, squashing, angular velocity) then the SdW coefficients of the 4d bulk theory are constrained to obey

$$c^{\text{tot}} = b_1^{\text{tot}} = b_2^{\text{tot}} = 0 .$$

This is a strong constraint for the UV consistency of EFTs in AdS_4 !

A new tool to delineate the landscape of scale separated AdS_4 vacua?

Black holes and thermal observables

[Witten]; [Horowitz,Myers]; [NPB,Charles,Hristov,Reys]; [NPB,Hong,Reys];
[Iliesiu,Kolođlu,Mahajan,Perlmutter,Simmons-Duffin]; [Luo,Wang]; [Benjamin, Lee,Ooguri,Simmons-Duffin]

BHs and thermal observables

Using the results above we can also compute the leading corrections to the entropy of **any** large asymptotically $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ black hole.

Example: AdS-Schwarzschild black hole

$$S_{\text{Sch}}^{\text{ABJM}} = \frac{2\pi r_+^2}{L^2} \frac{\sqrt{2k}}{3} \left(N^{\frac{3}{2}} + \frac{16 - k^2}{16k} N^{\frac{1}{2}} \right) + \frac{2\pi}{\sqrt{2k}} N^{\frac{1}{2}} - \frac{1}{2} \log N$$

Consider a 3d CFT on $S^1_\beta \times \mathbb{R}^2$. The vev of the stress-energy tensor and the thermal free energy are

$$\langle \mathcal{T}^{00} \rangle = \frac{2}{3} \frac{b_{\mathcal{T}}}{\beta^3}, \quad F_{S^1_\beta \times \mathbb{R}^2} = \frac{f_{\mathcal{T}}}{\beta^3}, \quad 3f_{\mathcal{T}} = b_{\mathcal{T}}.$$

To compute $f_{\mathcal{T}}$ in the bulk use the “AdS soliton”. For the ABJM theory we find

$$b_{\mathcal{T}} = -\frac{8\pi^2 \sqrt{2k}}{27} N^{\frac{3}{2}} + \frac{\pi^2 (k^2 - 16)}{27\sqrt{2k}} N^{\frac{1}{2}} + 0 \times \log N \dots$$

Somewhat surprisingly we find that to this order at large N $b_{\mathcal{T}} = -\frac{\pi^3}{72} C_{\mathcal{T}}!$

Summary

- Presented exact results for the large N limit of the partition function of the ABJM theory on S^3 , $S^1 \times \Sigma_g$, and $S^1 \times_{\omega} S^2$ focusing on the $\log N$ contribution.
- Discussed how these log terms can be reproduced by supergravity and string/M-theory via AdS/CFT.
- Important for understanding the entropy of supersymmetric AdS_4 Reissner-Nordström and Kerr-Newman black holes.
- New constraints on gravity + EFTs in AdS?
- Application of these results to non-supersymmetric black hole thermodynamics and CFT thermal observables.

Outlook

Results I did not discuss

- Large N supersymmetric partition functions for other 3d $\mathcal{N} = 2$ holographic SCFTs via supersymmetric localization.
- Similar logarithmic correction results for the holographically dual AdS_4 backgrounds in string/M-theory.
- Similar large N and holographic results for 3d $\mathcal{N} = 2$ SCFTs arising from M5-branes (class \mathcal{R} SCFTs).

Some open questions

- A better understanding of the simplicity and universality of the logarithmic corrections.
- Derivation from (or lessons for) type IIA string theory and M-theory?
- OSV-type conjecture for AdS black holes?
- Application of the “unbearable lightness” constraint to candidate scale separated AdS_4 vacua?

