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Holographic Wilson loop one-point functions in ABJM theory

Xiao-Yi Zhang,[a] **Yunfeng Jiang**[b,c,*] **and Jun-Bao Wu**[a,c,*]

[a]*Center for Joint Quantum Studies and Department of Physics, School of Science, Tianjin University, 135 Yaguan Road, Tianjin 300350, P.R. China*

[b]*School of physics and Shing-Tung Yau Center, Southeast University, 2 Sipailou, Nanjing 211189, P.R. China*

[c]*Peng Huanwu Center for Fundamental Theory, 96 Jinzhai Road, Hefei, Anhui 230026. P.R. China*

E-mail: xiaoyi_zhang@tju.edu.cn, jinagyf2008@seu.edu.cn, junbao.wu@tju.edu.cn

Abstract: We compute the correlation function between a circular half-BPS Wilson loop (or straight Wilson line) and a local operator in ABJM theory utilizing its M-theory description. The local operator can be a 1/3-BPS single-trace chiral primary operator or the stress-energy tensor. Using the AdS/CFT correspondence, these correlators are dual to fluctuations of a probe M2-brane in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. We derive analytic results for both cases and compare them with existing results based on supersymmetric localization in the literature. In the large-N limit with k finite, our holograkphic results exhibit perfect agreement with localization.

Keywords: AdS-CFT Correspondence, Chern-Simons Theories, M-Theory, Wilson, 't Hooft and Polyakov loops

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The unusual ordering of authors instead of the standard alphabetical one in hep-th community is for students to get proper recognition of contribution under the current out-dated practice in China.

*Corresponding author.

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1 Introduction

Conformal field theories (CFTs) can be enriched by the inclusion of extended objects such as line and surface defects. Among these, Wilson lines and loops (WLs) have been the most extensively studied due to their rich physical and mathematical properties. In supersymmetric quantum field theories (SQFTs), certain classes of WLs preserve enough supersymmetry to allow for exact computations, making them invaluable probes of non-perturbative dynamics. The study of WLs has led to significant advances through the interplay of diverse non-perturbative methods, including holography [1–10], supersymmetric localization [11–16], the conformal bootstrap [17–21], integrability [16, 22–33], and *bootstrability* (the combination of the last two tools) [34–36]. Moreover, WLs play a central role in the AdS/CFT correspondence, where they correspond to fundamental strings [1, 2]/D-branes [3, 4] in the bulk or certain bubbling geometries [37–39], thereby enriching the holographic dictionary [40–42].

The investigation of WLs in $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory has a long and fruitful history. A non-perturbative result in terms of matrix model integral was proposed based on the structure of perturbative results [43], confirming previous holographic calculations at strong coupling [44, 45]. A major breakthrough came with the application of supersymmetric localization, which reduced the computation of WL expectation values to a matrix model [11], which is the same one conjectured from the perturbative computations. This framework was subsequently extended to less supersymmetric WLs [5–10, 13, 46]. In contrast, WLs in the three-dimensional $\mathcal{N} = 6$ ABJM theory [47] have received comparatively less attention. So far, the focus has been on the bosonic 1/6-BPS loop [48–50] and the 1/2-BPS loops [51].[1] The localization of these WLs leads to a highly non-trivial matrix models [12], which are much more challenging to evaluate and thus spurred the development of the Fermi gas approach to

[1] A nice review on various aspects of BPS WLs in ABJM theory is [52].

evaluate the resulting matrix integrals [53, 54]. The holographic duals of some WLs have been explored in [48, 50], where they were identified with (smeared) string and D-brane configurations in $\mathrm{AdS}_4 \times \mathbb{CP}^3$, or (smeared) membrane and other objects in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. We recall that ABJM theory has two kinds of large N limit,

- 't Hooft limit: take $N, k \to \infty$ with the 't Hooft coupling $\lambda = N/k$ fixed. Large λ limit in this case is dual to type IIA superstring theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$.
- M-theory limit: take $N \to \infty$ with k kept finite. The holographic description is in terms of M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$.

A CFT coupled to extended objects is referred to as a defect CFT (dCFT), where the defect introduces additional observables, such as correlation functions in the presence of the defect. Among the simplest yet most important observables in a dCFT are defect one-point functions, which encode the operator product expansion (OPE) coefficients of the WL. Physically, these coefficients describe how a small WL can be approximated by a local operator insertion,[2] making them fundamental data for characterizing the defect [44, 55]. While such OPE coefficients are generally challenging to compute, progress has been made in certain cases using bootstrap techniques [55, 56], integrability [33], localization [14, 15], and holography [44, 57, 58].

In this paper, we study WL one-point functions in ABJM theory, focusing on the cases where the WL is 1/2-BPS and either an infinite straight line or a circular loop. The local operators under consideration are BPS operators: the stress-energy tensor and a 1/3-BPS chiral primary operator (CPO). We compute these correlation functions holographically by evaluating specific fluctuations of the classical membrane configuration in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. Beyond their intrinsic physical interest, such OPE coefficients in the M-theory limit remain scarce in the literature, making our results a valuable addition to the strong coupling data for studying defect CFTs. Moreover, our approach provides a solid basis for computing similar observables of other types of defect correlation functions in ABJM theory.

To validate our holographic results, we compare them with existing localization computations for these correlators [59–61]. By taking the M-theory limit, we find agreement between the two approaches. In the case of CPO, the localization result is only available for the operator with conformal dimension $\Delta = 1$, which we compared with the holographic result and find a perfect match. Notably, the matching requires careful normalization of the OPE coefficients, particularly for the CPO, where the two-point function normalization is a highly non-trivial function of the gauge group rank N and the Chern-Simons level k (Two-loops correction to this normalization factor in the field theory side were computed in [62, 63]).

The remainder of this paper is organized as follows. In section 2, we outline the general setup for WL one-point functions, detailing the relevant operators and the constraints imposed by conformal symmetry. Conformal invariance fixes the spacetime dependence of these correlators, reducing the problem to the computation of a single OPE coefficient. In section 3, we present the holographic computation of these coefficients. In section 4, we

[2]More precisely, we mean that the size of the WL is much smaller than the distance between it and the local operator. This corresponds to the limit $R \ll L$ in the right panel of figure 1.

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compare our results with localization predictions, verifying their consistency in the M-theory limit. We conclude in section 5 with a discussion of future directions.

2 Correlators of a 1/2-BPS Wilson loop and a local operator

In this section, we outline the setup for computing the correlation functions of interest in ABJM theory, which is a three-dimensional $\mathcal{N}=6$ super-Chern-Simons theory with gauge group $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$. The theory contains two gauge fields A_μ and $\hat{A}_\mu$, together with four scalars Y^I and four Dirac spinors ψ_I in the bi-fundamental representation of the gauge group. We work in Euclidean $\mathbb{R}^3$ with coordinates $x^\mu = (x^1, x^2, x^3)$.[3]

Half-BPS Wilson line/loop. We consider a half-BPS Wilson line along the x^1-axis, localized at $x^2 = x^3 = 0$. The contour L is parameterized as

$$x^\mu(\tau) = (\tau, 0, 0)\,,\ -\infty < \tau < \infty\,. \tag{2.1}$$

The Wilson line in the fundamental representation of the supergroup $\mathrm{U}(N|N)$ is given by

$$W[L] = \mathrm{Tr}\mathcal{P}\,\exp\left(-\mathrm{i}\int_L d\tau \mathcal{L}^L_{1/2}(\tau)\right)\,, \tag{2.2}$$

where the superconnection $\mathcal{L}^L_{1/2}$ takes the form

$$\mathcal{L}^L_{1/2} = \begin{pmatrix} \mathcal{A}^L & \bar{f}^L_1 \\ f^L_2 & \hat{\mathcal{A}}^L \end{pmatrix}\,. \tag{2.3}$$

The components are explicitly:

$$\mathcal{A}^L = A_\mu \dot{x}^\mu + \frac{2\pi\mathrm{i}}{k}(\delta^J_I - 2\alpha_I\bar{\alpha}^J)Y^I\bar{Y}_J|\dot{x}|\,, \tag{2.4}$$

$$\hat{\mathcal{A}}^L = \hat{A}_\mu \dot{x}^\mu + \frac{2\pi\mathrm{i}}{k}(\delta^J_I - 2\alpha_I\bar{\alpha}^J)\bar{Y}_J Y^I|\dot{x}|\,, \tag{2.5}$$

$$\bar{f}^L_1 = \sqrt{\frac{2\pi}{k}}\bar{\alpha}^I u_+ \psi_I |\dot{x}|\,, \tag{2.6}$$

$$f^L_2 = \sqrt{\frac{2\pi}{k}}\bar{\psi}^I u_- \alpha_I |\dot{x}|\,. \tag{2.7}$$

Here, $\dot{x}^\mu = dx^\mu/d\tau$, and α_I are complex parameters satisfying $\sum_{I=1}^4 \alpha_I\bar{\alpha}^I = 1$, with $\bar{\alpha}^I = (\alpha_I)^*$. The spinors $u_\pm$ follow the of [64] and are given by

$$u^\alpha_\pm = (\mp\mathrm{i}, -1)\,, \quad u_{\pm\alpha} = \begin{pmatrix} 1 \\ \mp\mathrm{i} \end{pmatrix}\,. \tag{2.8}$$

We also consider the following half-BPS circular Wilson loop along the circle $C = \{x^\mu = (R\cos\tau, R\sin\tau, 0)|\tau \in [0, 2\pi]\}$,

$$W[C] = \mathrm{Tr}\mathcal{P}\,\exp\left(-\mathrm{i}\oint_C d\tau \mathcal{L}^C_{1/2}(\tau)\right)\,, \tag{2.9}$$

[3]We use $\mu, \nu, \cdots$ to denote coordinates of $\mathbb{R}^3$, $M, N, \cdots$ coordinates of $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$, $m, n, \cdots$ coordinates of AdS_4, $\alpha, \beta, \cdots$ coordinates of $S^7/\mathbb{Z}_k$, and finally $a, b, \cdots$ coordinates of the worldvolume of the probe M2-brane.

with the superconnection given by

$$\mathcal{L}^C_{1/2} = \begin{pmatrix} \mathcal{A}^C & \bar{f}^C_1 \\ f^C_2 & \hat{\mathcal{A}}^C \end{pmatrix} . \tag{2.10}$$

The components explicitly read:

$$\mathcal{A}^C = A_\mu \dot{x}^\mu - \mathrm{i}\frac{2\pi}{k}(2\alpha_I \bar{\alpha}^J - \delta^J_I)|\dot{x}|Y^I \bar{Y}_J \,, \tag{2.11}$$

$$\hat{\mathcal{A}}^C = \hat{A}_\mu \dot{x}^\mu - \mathrm{i}\frac{2\pi}{k}(2\alpha_I \bar{\alpha}^J - \delta^J_I)|\dot{x}|\bar{Y}_J Y^I \,, \tag{2.12}$$

$$\bar{f}^C_1 = -\sqrt{\frac{2\pi}{k}}\bar{\alpha}^I \bar{\zeta}\psi_I |\dot{x}| \,, \tag{2.13}$$

$$f^C_2 = \sqrt{\frac{2\pi}{k}}\bar{\psi}^I \eta \alpha_I |\dot{x}| \,, \tag{2.14}$$

where α_I and $\bar{\alpha}^I$ are the same as the Wilson line case, and the spinors $\bar{\zeta}^\alpha$ and η_α are

$$\bar{\zeta}^\alpha = \left(e^{\mathrm{i}\tau/2}, e^{-\mathrm{i}\tau/2}\right) , \quad \eta_\alpha = \begin{pmatrix} e^{-\mathrm{i}\tau/2} \\ e^{\mathrm{i}\tau/2} \end{pmatrix} . \tag{2.15}$$

Local operators. We are interested in the correlation functions of the Wilson line/loop and a local operator. In this work, we will consider two types of local operators: 1) The stress energy tensor of the theory; 2) The 1/3-BPS chiral primary operator.

Our choice of the normalization of the stress energy tensor follows the one in [59],

$$T_{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\partial(\sqrt{g}\mathcal{L})}{\partial g^{\mu\nu}}\bigg|_{g_{\mu\nu}=\delta_{\mu\nu}} , \tag{2.16}$$

where $\mathcal{L}$ is the Lagrangian density of the theory. The definition of the 1/3-BPS CPO is

$$\mathcal{O}^A = (C^A)^{J_1\cdots J_L}_{I_1\cdots I_L}\mathrm{Tr}(Y^{I_1}\bar{Y}_{J_1}\cdots Y^{I_L}\bar{Y}_{J_L}) \,, \tag{2.17}$$

with C^A being a symmetric traceless tensor, satisfying

$$(C^A)^{J_1\cdots J_L}_{I_1\cdots I_L} = (C^A)^{J_1\cdots J_L}_{(I_1\cdots I_L)} = (C^A)^{(J_1\cdots J_L)}_{I_1\cdots I_L} \tag{2.18}$$

and

$$\delta^{I_1}_{J_1}(C^A)^{J_1\cdots J_L}_{I_1\cdots I_L} = 0 \,. \tag{2.19}$$

We also demand that C^A is normalized

$$(C^A)^{J_1\cdots J_L}_{I_1\cdots I_L}(\bar{C}^B)^{I_1\cdots I_L}_{J_1\cdots J_L} = \delta^{AB} \,, \tag{2.20}$$

where $\bar{C}^B$ is the complex conjugate of C^B.

For the case of a Wilson line, we place the local operator away from the line, as is shown in the left panel of figure 1. For the case of a Wilson loop, we place the operator far away from the Wilson loop, which is also known as the OPE limit. This is shown in the right panel of figure 1.

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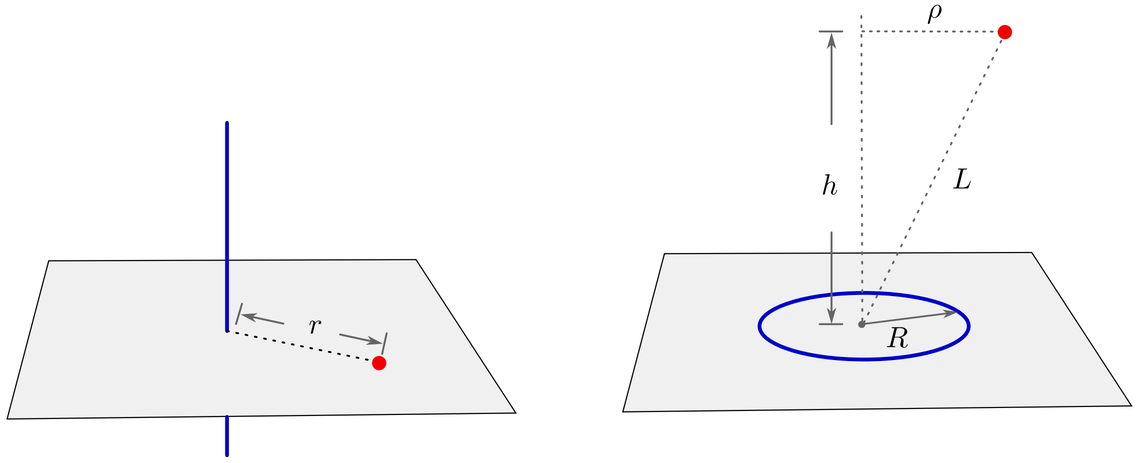

(a) Wilson line $\qquad\qquad$ (b) Wilson loop

Figure 1. Correlation function of a Wilson line/loop and a local operator. The blue line in the left and right panel stands for a Wilson line and Wilson loop, respectively. The red bullet denote a local operator. In the case of the Wilson loop, we consider the OPE limit $L \gg R$.

OPE coefficients. The spacetime dependence of the correlation function is fixed by conformal symmetry up to a constant. For the Wilson line one-point function of a CPO, we have

$$\frac{\langle W[L]\mathcal{O}(r)\rangle}{\langle W[L]\rangle} = \frac{B_{\mathcal{O}}}{r^{\Delta}} \tag{2.21}$$

where r is the distance between the local operator and the Wilson line and Δ is the scaling dimension of the local operator. We will compute the coefficient $B_{\mathcal{O}}$. In order to compare the results from localization and holography, we further divide $B_{\mathcal{O}}$ by a proper normalization of the two-point function and consider

$$\frac{\langle W[L]\mathcal{O}(r)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L]\rangle} = \frac{B_{\mathcal{O}}}{\sqrt{\mathcal{N}_{\mathcal{O}}}}\frac{1}{r^{\Delta}}\,. \tag{2.22}$$

For the circular Wilson loop with contour $x(\tau) = (R\cos\tau, R\sin\tau, 0)$ and a local operator at a generic point (y_1, y_2, y_3), conformal symmetry fixes the correlation function to be of the form [44, 65]

$$\frac{\langle W[C]\mathcal{O}(y)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[C]\rangle} = C_{\mathcal{O}}\frac{R^{\Delta}}{((h^2+\rho^2-R^2)^2+4R^2h^2)^{\Delta/2}}\,, \tag{2.23}$$

where $\rho = \sqrt{y_1^2+y_2^2}$, $h = |y_3|$ and $C_{\mathcal{O}}$ is a constant. In the OPE limit $L = \sqrt{\rho^2+h^2} \gg R$, we have

$$\frac{\langle W[C]\mathcal{O}(y)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[C]\rangle} = C_{\mathcal{O}}\frac{R^{\Delta}}{L^{2\Delta}}\,. \tag{2.24}$$

The normalization factor $\mathcal{N}_{\mathcal{O}}$ here is the same as the one in (2.22). The coefficients $B_{\mathcal{O}}$ and $C_{\mathcal{O}}$ are related by [59]

$$\frac{B_{\mathcal{O}}}{\sqrt{\mathcal{N}_{\mathcal{O}}}} = \frac{C_{\mathcal{O}}}{2^{\Delta}}\,. \tag{2.25}$$

We will review the derivation of this relation in subsection 4.2.

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For a Wilson line $W[L]$ at $x_2 = x_3 = 0$ (extended in x_1 direction) and stress tensor $T_{\mu\nu}$ located at (y_1, y_2, y_3) [66], conformal symmetry fixes the correlation function of different components of the stress tensor to be

$$\begin{aligned}
\frac{\langle W[L] T_{11}(y)\rangle}{\langle W[L]\rangle} &= -\frac{h_W}{r^3}\,, \\
\frac{\langle W[L] T_{\kappa\sigma}(y)\rangle}{\langle W[L]\rangle} &= -\frac{h_W}{r^3}(3n_\kappa n_\sigma - 2\delta_{\kappa\sigma})\,, \\
\frac{\langle W[L] T_{1\kappa}(y)\rangle}{\langle W[L]\rangle} &= 0\,,
\end{aligned} \tag{2.26}$$

where $\kappa, \sigma = 2, 3$ and we have defined

$$r = \sqrt{y_2^2 + y_3^2}\,, \qquad n_\kappa = \frac{y_\kappa}{r}\,. \tag{2.27}$$

Dynamical information is contained in h_W, which is a non-trivial function of N and k.

Here a comment on normalization is in order. In contrast to equation (2.22), the Wilson loop one-point function of $T_{\mu\nu}$ is not normalized by the two-point function of $T_{\mu\nu}$. This is because the normalization of the stress tensor, as defined in (2.16), has a clear physical meaning: the normalization of its two-point correlator yields the stress-tensor central charge [67], which measures the degrees of freedom in the field theory.

3 Holographic computations

In this section, we compute the Wilson line/loop OPE coefficients by using holography. According to the AdS/CFT correspondence, ABJM theory with large N and finite k is dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. Our calculation captures the leading-order result in the large N limit while keeping k fixed. The eleven-dimensional supergravity background consists of the metric

$$ds^2 = g_{MN} dX^M dX^N = ds^2_{\mathrm{AdS}_4} + 4 ds^2_{S^7/\mathbb{Z}_k}\,, \tag{3.1}$$

and the four-form flux

$$F_4 = 3\Omega_{\mathrm{AdS}_4} \tag{3.2}$$

where we have set the radius of the AdS_4 to 1, and Ω_{AdS_4} denotes the volume form of the unit AdS_4. From the AdS/CFT dictionary, the eleven-dimensional Planck length is given by

$$l_p = \left(\frac{2}{\pi^2 k N}\right)^{1/6}. \tag{3.3}$$

Euclidean AdS_4 metric. The metric on the unit (Euclidean) AdS_4 ($E\mathrm{AdS}_4$) in Poincaré coordinates is

$$ds^2_{\mathrm{AdS}_4} = \frac{1}{z^2}(dz^2 + \delta_{ij} dx^i dx^j)\,, \tag{3.4}$$

where $i, j = 1, 2, 3$.

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Parametrization of $S^7/\mathbb{Z}_k$. To describe the metric on $S^7/\mathbb{Z}_k$, we parameterize the space using four complex coordinates $z^I, (I = 1, \cdots, 4)$ subject to the constraint $\sum_{I=1}^4 |z^I|^2 = 1$. Explicitly,

$$z^1 = \cos\xi \cos\frac{\theta_1}{2} \exp\left[\mathrm{i}\left(\zeta + \frac{\psi + \varphi_1}{2}\right)\right], \tag{3.5}$$

$$z^2 = \cos\xi \sin\frac{\theta_1}{2} \exp\left[\mathrm{i}\left(\zeta + \frac{\psi - \varphi_1}{2}\right)\right], \tag{3.6}$$

$$z^3 = \sin\xi \cos\frac{\theta_2}{2} \exp\left[\mathrm{i}\left(\zeta + \frac{-\psi + \varphi_2}{2}\right)\right], \tag{3.7}$$

$$z^4 = \sin\xi \sin\frac{\theta_2}{2} \exp\left[\mathrm{i}\left(\zeta + \frac{-\psi - \varphi_2}{2}\right)\right], \tag{3.8}$$

with the coordinate ranges:

$$0 \le \xi \le \frac{\pi}{2}, \quad -\pi \le \psi \le \pi, \quad 0 \le \theta_i \le \pi, \quad 0 \le \varphi_i \le 2\pi, \quad (i = 1, 2), \tag{3.9}$$

and the angular identification $\zeta \sim \zeta + \frac{2\pi}{k}$ due to the $\mathbb{Z}_k$ quotient. The metric on $S^7/\mathbb{Z}_k$, induced from the flat metric $ds_{\mathbb{C}^4} = \sum_{i=1}^4 dz^I d\bar{z}_I$, takes the form

$$ds_{S^7/\mathbb{Z}_k} = (d\zeta + A)^2 + ds_{\mathbb{CP}^3}, \tag{3.10}$$

where the one-form A is given by

$$A = \frac{1}{2}\cos(2\xi)d\psi + \frac{1}{2}\cos^2\xi \cos\theta_1 d\varphi_1 + \frac{1}{2}\sin^2\xi \cos\theta_2 d\varphi_2, \tag{3.11}$$

and the Fubini-Study metric on $\mathbb{CP}^3$ is

$$\begin{aligned} ds_{\mathbb{CP}^3} = d\xi^2 &+ \frac{1}{4}\cos^2\xi(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2) + \frac{1}{4}\sin^2\xi(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2) \\ &+ \cos^2\xi \sin^2\xi \left(d\psi + \frac{1}{2}\cos\theta_1 d\varphi_1 - \frac{1}{2}\cos\theta_2 d\varphi_2\right). \end{aligned} \tag{3.12}$$

Four-form flux. In Poincaré coordinates, the four-form flux F_4 and its corresponding three-form potential c_3 are

$$\begin{aligned} F_4 &= \frac{3}{z^4} dz \wedge dx^1 \wedge dx^2 \wedge dx^3, \\ c_3 &= -\frac{1}{z^3} dx^1 \wedge dx^2 \wedge dx^3. \end{aligned} \tag{3.13}$$

3.1 Holographic description of the operators

We now employ the M-theory framework to compute the normalized correlation function of a Wilson loop operator $W[L]$ or $W[C]$ and a local operator. This framework is dual to ABJM theory in the large N limit with k fixed. Notice that this is different from the 't Hooft limit which is $N, k \to \infty$ with their ratio N/k, known as the 't Hooft coupling λ, fixed.

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The Wilson line. In the holographic dual, the WL is described by a probe M2-brane embedded in the eleven-dimensional supergravity background $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. The bosonic part of the M2-brane action consists of the Dirac-Born-Infeld (DBI) term and the Wess-Zumino (WZ) term:[4]

$$S_{\mathrm{M2}} = T_{\mathrm{M2}} \left(\int d^3\sigma \sqrt{\det \tilde{g}} - P[c_3] \right) \tag{3.14}$$

where $\tilde{g}_{ab} = \partial_a X^M \partial_b X^N g_{MN}$ is the induced metric on the M2-brane worldvolume and $P[c_3]$ is the pullback of c_3 to the membrane worldvolume. Here T_{M2} is the tension of the M2-brane

$$T_{\mathrm{M2}} = \frac{1}{4\pi^2 l_p^3} \,. \tag{3.15}$$

For the computation of WL one-point functions in this paper, the Wess-Zumino term does not contribute, as the M2-brane wraps two directions in AdS_4 and one direction in $S^7/\mathbb{Z}_k$.

The holographic description of $W[L]$ (in the fundamental representation of the supergroup $\mathrm{U}(N|N)$) is in terms of the following probe M2-brane in $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. Denoting the worldvolume coordinates of this M2-brane as $(\sigma^0, \sigma^1, \sigma^2)$, the embedding of this brane is

$$\begin{aligned} &z = \sigma^0 \,, \qquad x_1 = \sigma^1 \,, \quad x_2 = x_3 = 0 \,, \\ &\zeta = \sigma^2 + \zeta^0 \,, \quad \xi = \xi^0 \,, \quad \theta_1 = \theta_1^0 \,, \quad \varphi_1 = \varphi_1^0 \,, \quad \theta_2 = \theta_2^0 \,, \quad \varphi_2 = \varphi_2^0 \,, \quad \psi = \psi^0 \,, \end{aligned} \tag{3.16}$$

where $\sigma^0 \geq 0, \sigma^1 \in (-\infty, \infty), \sigma^2 \in [0, \frac{2\pi}{k}]$, and $\zeta^0, \xi^0, \theta_1^0, \varphi_1^0, \theta_2^0, \varphi_2^0, \psi^0$ are constants. The worldvolume of this M2-brane has the topology $\mathrm{AdS}_2 \times S^1$ with AdS_2 in AdS_4 and S^1 along the ζ-cycle. Notice that the above AdS_2 is in the Poincaré coordinates, and the ζ-cycle is the M-theory circle when we consider reduction of the M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ to the IIA superstring theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$. The parameters α_I defining the Wilson line in the boundary theory are related to the brane embedding via [68]

$$\alpha_I = \bar{z}_I(\xi^0, \theta_1^0, \varphi_1^0, \theta_2^0, \varphi_2^0, \zeta^0) \,. \tag{3.17}$$

The circular Wilson loops. The M2-brane dual to circular Wilson loops has worldvolume $E\mathrm{AdS}_2 \times S^1$ with $E\mathrm{AdS}_2$ now in the global coordinates. Let us denote the worldvolume coordinates of the M2-brane by $(\sigma^0, \sigma^1, \sigma^2)$. The map $X^m(\sigma)$ from the membrane worldvolume to the AdS_4 part of the background geometry is specified by:

$$z = R \sin \sigma^1 \,, \quad x_1 = R \cos \sigma^1 \cos \sigma_0 \,, \quad x_2 = R \cos \sigma^1 \sin \sigma^0 \,, \quad x_3 = 0 \,. \tag{3.18}$$

The map $X^\alpha(\sigma)$ from the worldvolume to the $S^7/\mathbb{Z}_k$ part and the relation between the embedding and the parameters α_I is the same as the previous Wilson line case. The worldvolume coordinate ranges are

$$0 \leq \sigma^0 \leq 2\pi, \quad 0 \leq \sigma^1 \leq \frac{\pi}{2}, \quad 0 \leq \sigma^2 \leq \frac{2\pi}{k} \,, \tag{3.19}$$

[4] Since we only use the bosonic part, it will be referred to as the membrane action or M2-brane action in the following.

The local operators. In the holographic framework, the local operators we consider correspond to perturbations of the background metric and flux fields.

According to the AdS_4/CFT_3 duality, the boundary stress tensor $T_{\mu\nu}$ is dual to metric fluctuations in the AdS_4 subspace of the full $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ background. The perturbed metric takes the form

$$G_{mn} = g_{mn} + \delta g_{mn}\,, \tag{3.20}$$

where g_{mn} is the unperturbed AdS_4 metric. The explicit expression for δg_{mn} will be provided in the next subsection when we perform concrete calculations.

The chiral primary operators $\mathcal{O}^A$ are dual to fluctuations of both the metric and the three-form potential in the eleven-dimensional background:

$$G_{MN} = g_{MN} + \delta g^A_{MN}\mathbf{Y}^A\,, \tag{3.21}$$

$$C_{MNP} = c_{MNP} + \delta c^A_{MNP}\mathbf{Y}^A\,, \tag{3.22}$$

where $\mathbf{Y}^A = (C^A)^{J_1\cdots J_L}_{I_1\cdots I_L}\alpha_{J_1}\cdots\alpha_{J_L}\bar{\alpha}^{I_1}\cdots\bar{\alpha}^{I_L}$ are spherical harmonics of $S^7/\mathbb{Z}_k$ with radius 1. The explicit form of the fluctuations will be given when we carry out explicit computations in the following sections.

3.2 Holographic calculations of $\langle W[L]T_{\mu\nu}\rangle/\langle W[L]\rangle$

We compute the variation of the M2-brane action, δS_{M2}, induced by fluctuations of the background metric in eq. (3.20). From this, we derive the correlation function between the Wilson loop $W[L]$ and the stress tensor $T_{\mu\nu}(y)$ as[5]

$$\frac{\langle W[L]T_{\mu\nu}(y)\rangle}{\langle W[L]\rangle} = -2\frac{\delta S_{\mathrm{DBI}}}{\delta\hat{h}^{\mu\nu}(y)}\,. \tag{3.23}$$

Here a key comment is in order. As stressed in [70, 71], in holographic computations of three-point functions involving classical string/brane solutions, we should average over classical configurations of the strings/branes. This is equivalent to averaging over the orbit generated by the broken global symmetries acting on the strings/branes. However, in the case at hand and that considered below, the membranes (and the dual Wilson loops) are coherent states instead of eigenstates with respect to the global charges broken by the membrane solutions, in the sense of [70]. So no orbit average is needed here.

We now go back to the concrete calculations. The explicit form of the metric fluctuation δg_{mn} in (3.20) is given by [72][6]

$$\delta g_{mn}(z,x) = \kappa_3 z\int d^3y\frac{1}{f^3}J_{m\mu}(X-Y)\,J_{n\nu}(X-Y)\,\mathcal{P}^{\mu\nu,\,\rho\sigma}\hat{h}_{\rho\sigma}(y)\,, \tag{3.24}$$

where $X=(z,x)$ denotes a bulk point in AdS_4 and $Y=(0,y)$ is a point at the boundary. $\hat{h}_{\rho\sigma}(y)$ is the fluctuation of the metric on the boundary of AdS_4 which couples to the stress

[5]The factor 2 in the eq. (3.23) was explained in [69].

[6]The indices of J_{mn} and $\mathcal{P}_{\mu\nu,\,\rho\sigma}$ are always contracted with the flat metric.

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tensor $T^{\rho\sigma}(y)$. The functions $f, J_{mn}, \mathcal{P}_{\mu\nu,\rho\kappa}$ are defined as[7]

$$f := |X - Y|^2 := z^2 + (x-y)^2\,, \tag{3.25}$$

$$J_{mn}(X) := \delta_{mn} - 2\frac{X_m X_n}{|X|^2}\,, \tag{3.26}$$

$$\mathcal{P}^{\mu\nu,\,\rho\sigma} := \frac{1}{2}(\delta^{\mu\rho}\delta^{\nu\sigma} + \delta^{\mu\sigma}\delta^{\nu\rho}) - \frac{1}{3}\delta^{\mu\nu}\delta^{\rho\sigma}\,. \tag{3.27}$$

The constant κ_d for general d is given by

$$\kappa_d = \frac{d+1}{d-1}\frac{\Gamma[d]}{\pi^{d/2}\Gamma(d/2)}\,, \tag{3.28}$$

yielding $\kappa_3 = 8/\pi^2$ (corresponding to $d=3$) for the case at hand. The induced metric components are

$$\tilde{g}_{00} = \frac{1}{(\sigma^0)^2}\,, \qquad \tilde{g}_{11} = \frac{1}{(\sigma^0)^2}\,, \qquad \tilde{g}_{22} = 4\,. \tag{3.29}$$

Expanding eq. (3.24) explicitly, we obtain

$$\begin{aligned}\delta g_{zz} &= \frac{8}{\pi^2}\int d^3y \frac{z}{f^3}\left[\delta_{\mu z} - \frac{2z(x-y)_\mu}{f}\right]\left[\delta_{\nu z} - \frac{2z(x-y)_\nu}{f}\right] \\ &\quad\times\left(\frac{1}{2}\delta^{\mu\rho}\delta^{\nu\sigma} + \frac{1}{2}\delta^{\mu\sigma}\delta^{\nu\rho} - \frac{1}{3}\delta^{\mu\nu}\delta^{\rho\sigma}\right)\hat{h}_{\rho\sigma}(y) \\ &= \frac{32}{\pi^2}\int d^3y \frac{z^3(x-y)_\mu(x-y)_\nu}{f^5}\left(\frac{1}{2}\delta^{\mu\rho}\delta^{\nu\sigma} + \frac{1}{2}\delta^{\mu\sigma}\delta^{\nu\rho} - \frac{1}{3}\delta^{\mu\nu}\delta^{\rho\sigma}\right)\hat{h}_{\rho\sigma}(y)\,,\end{aligned} \tag{3.30}$$

and

$$\begin{aligned}\delta g_{x^1x^1} &= \frac{8}{\pi^2}\int d^3y \frac{z}{f^3}\left[\delta_{\mu x^1} - \frac{2(x-y)_1(x-y)_\mu}{f}\right] \\ &\quad\times\left[\delta_{\nu x^1} - \frac{2(x-y)_1(x-y)_\nu}{f}\right]\left(\frac{1}{2}\delta^{\mu\rho}\delta^{\nu\sigma} + \frac{1}{2}\delta^{\mu\sigma}\delta^{\nu\rho} - \frac{1}{3}\delta^{\mu\nu}\delta^{\rho\kappa}\right)\hat{h}_{\rho\sigma}(y)\,.\end{aligned} \tag{3.31}$$

Now we proceed to compute different components of the correlation function (3.23).

- For $\mu = \nu = 1$, the relevant metric variations are

$$\int d^3\sigma \frac{\delta g_{zz}}{\delta \hat{h}_{11}(y)} = \frac{32}{\pi^2}\int d^3\sigma \frac{z^3}{f^5}\left(\frac{2}{3}x_1^2 - \frac{1}{3}r^2\right)$$

  and

$$\int d^3\sigma \frac{\delta g_{x_1x_1}}{\delta \hat{h}_{11}(y)} = \frac{8}{\pi^2}\int d^3\sigma \frac{z}{f^3}\left[\left(1 - \frac{2x_1^2}{f}\right)^2 - \frac{1}{3}\left(1 - \frac{4x_1^2}{f} + \frac{4x_1^2(x_1^2+r^2)}{f^2}\right)\right]\,,$$

  where $y = (0, y_2, y_3)$, and we have defined $r := \sqrt{y_2^2 + y_3^2}, n_\kappa := y_\kappa/r$, for $\kappa = 2, 3$. The correlation function evaluates to

$$\begin{aligned}\frac{\langle W[L]T_{11}(y)\rangle}{\langle W[L]\rangle} &= -\frac{128}{\pi^2}T_{\mathrm{M2}}\int_0^{\frac{2\pi}{k}} d\zeta \int_0^\infty dz \int_{-\infty}^\infty dx_1 \frac{z[r^4 - 4r^2x_1^2 + (x_1^2+z^2)^2]}{12(r^2+x_1^2+z^2)^5} \\ &= -\frac{T_{\mathrm{M2}}}{kr^3} = -\frac{1}{4\sqrt{2}\pi}\sqrt{\frac{k}{N}}\frac{1}{r^3}\,,\end{aligned} \tag{3.32}$$

  where we have used (3.3) and (3.15).

[7] The definition of X_m is $X_m := \delta_{mn}X^n$.

- For $\mu = 1$, $\nu = 2, 3$, the metric variations vanish:

$$\int d^3\sigma \frac{\delta g_{zz}}{\delta \hat{h}^{1\nu}(y)} = \int d^3\sigma \int d^3y \frac{-4z^3 x_1 y^\nu}{f^5} = 0\,, \tag{3.33}$$

  and

$$\int d^3\sigma \frac{\delta g_{x_1x_1}}{\delta \hat{h}^{1\nu}(y)} = \int d^3\sigma \int d^3y \frac{z}{f^3}\left(1 - \frac{2x_1^2}{f}\right)\frac{2x_1 y^\nu}{f} = 0\,. \tag{3.34}$$

  Thus, the corresponding components of eq. (3.23) are zero.

- For $\mu, \nu = 2, 3$, the metric fluctuations take the form

$$\begin{aligned} \delta g_{zz} &= \int d^3y \frac{32z^3}{\pi^2 f^5}\left[r^2\left(n^\mu n^\nu - \frac{1}{3}\delta^{\mu\nu}\right) - \frac{1}{3}x_1^2\delta^{\mu\nu}\right]\hat{h}_{\mu\nu}(y)\,, \\ \delta g_{x^1x^1} &= -\int d^3y \frac{8z}{3\pi^2 f^3}\delta^{\mu\nu}\left[1 - \frac{4x_1^2}{f} + \frac{4x_1^2(x_1^2 + r^2)}{f^2}\right]\hat{h}_{\mu\nu}(y)\,, \end{aligned} \tag{3.35}$$

  leading to the correlation function

$$\frac{\langle W[L] T_{\mu\nu}(y)\rangle}{\langle W[L]\rangle} = -2\frac{\delta S_{\mathrm{M2}}}{\delta \hat{h}^{\mu\nu}(y)} = -\frac{1}{4\sqrt{2}\pi}\sqrt{\frac{N}{k}}\frac{1}{r^3}\left(3n^\mu n^\nu - 2\delta^{\mu\nu}\right)\,. \tag{3.36}$$

The results for the correlation function (3.23) can be summarized as

$$\frac{\langle W[L] T_{11}(y)\rangle}{\langle W[L]\rangle} = -\frac{h_W}{r^3}\,, \tag{3.37}$$

$$\frac{\langle W[L] T_{\kappa\sigma}(y)\rangle}{\langle W[L]\rangle} = -\frac{h_W}{r^3}(3n_\kappa n_\sigma - 2\delta_{\kappa\sigma})\,, \tag{3.38}$$

$$\frac{\langle W[L] T_{1\kappa}(y)\rangle}{\langle W[L]\rangle} = 0\,, \tag{3.39}$$

with

$$h_W = \frac{1}{4\sqrt{2}\pi}\sqrt{\frac{N}{k}} = \frac{\sqrt{\lambda}}{4\sqrt{2}\pi}\,, \tag{3.40}$$

and $\kappa, \sigma = 2, 3$. This structure agrees with symmetry considerations (2.26), with the dynamical information encoded in the scaling function h_W.

3.3 Holographic calculations of $\langle \mathcal{O}\rangle_{W[C]}$

In this subsection, we compute the normalized correlation function of the half-BPS circular Wilson loop $W[C]$ and the 1/3-BPS chiral primary operator $\mathcal{O}^A$

$$\langle \mathcal{O}^A(y)\rangle_{W[C]} := \frac{\langle \mathcal{O}^A(y) W[C]\rangle}{\langle W[C]\rangle\sqrt{\mathcal{N}_{\mathcal{O}^A}}}\,, \tag{3.41}$$

where the normalization factor $\mathcal{N}_{\mathcal{O}^A}$ is defined via the two-point function of $\mathcal{O}^A$

$$\langle \mathcal{O}^A(x)\mathcal{O}^{B\dagger}(y)\rangle = \frac{\delta^{AB}\mathcal{N}_{\mathcal{O}^A}}{|x-y|^{2\Delta_A}}\,, \tag{3.42}$$

with Δ_A being the conformal dimension of $\mathcal{O}^A$. We will focus on the OPE limit $L \gg R$ where L is the distance between the local operator and the center of the Wilson loop.

The CPO corresponds to fluctuations of the background fields in eqs. (3.21) and (3.22), with explicit forms given by [73–76]:

$$\delta g^A_{mn} = \frac{4}{J+2}\left[\nabla_m\nabla_n + \frac{J(J+6)}{8} g_{mn}\right] s^A - \frac{7J}{6} g_{mn} s^A\,, \tag{3.43}$$

$$\delta g^A_{\alpha\beta} = \frac{1}{3} J g_{\alpha\beta} s^A\,, \tag{3.44}$$

$$\delta c^A_{mnp} = 2\epsilon_{mnpq}\nabla^q s^A\,, \tag{3.45}$$

where $J = 2\Delta = 2L$. The scalar field $s^A(x,z)$ depends only on the coordinates (x,z) of AdS_4 and is determined by the boundary source $s^A_0(y)$

$$s^A(x,z) = \int d^3y G_\Delta(y;x,z) s^A_0(y)\,, \tag{3.46}$$

where $G_\Delta(y;x,z)$ is the boundary-to-bulk propagator

$$G_\Delta(y;x,z) = c\left(\frac{z}{z^2+|x-y|^2}\right)^\Delta\,, \tag{3.47}$$

with the constant c being [76]

$$c = 2^{\Delta-1}\pi l_p^4\sqrt{k}\frac{\Delta+1}{\Delta}\sqrt{2\Delta+1}\,. \tag{3.48}$$

In the OPE limit $L \gg R$, we can approximate

$$G_\Delta(y;x,z) \approx c\frac{z^\Delta}{\tilde{L}^{2\Delta}}\,, \tag{3.49}$$

$$\partial_z s^A \approx \frac{\Delta}{z} s^A\,, \tag{3.50}$$

$$\partial_\mu\partial_\nu s^A \approx \delta^z_\mu\delta^z_\nu\frac{\Delta(\Delta-1)}{z^2} s^A. \tag{3.51}$$

For Euclidean AdS_4 in the Poincaré coordinates, the Christoffel symbols are

$$\Gamma^z_{mn} = \frac{1}{2}g^{zz}(\partial_m g_{zn} + \partial_n g_{zm} - \partial_z g_{mn}) = z g_{mn} - \frac{2}{z}\delta^z_m\delta^z_n\,. \tag{3.52}$$

Substituting these into (3.43), the metric fluctuation simplifies to:

$$\delta g^A_{mn} = -\frac{2}{3}J g_{mn} s^A + \frac{J}{z^2}\delta^z_m\delta^z_n s^A\,. \tag{3.53}$$

The correlation function is computed from the variation of the M2-brane action due to background field fluctuations. As mentioned above, the Wess-Zumino term does not contribute, so the action variation is

$$\delta S_{\mathrm{M2}} = T_{\mathrm{M2}}\int d^3\sigma\sqrt{\det\tilde{g}_{ab}}\frac{1}{2}\tilde{g}^{ab}\partial_a X^M\partial_b X^N\delta g^A_{MN}\mathbf{Y}^A\,, \tag{3.54}$$

where the induced metric $\tilde{g}_{ab} = \partial_a X^M \partial_b X^N g_{MN}$ has components:

$$\tilde{g}_{00} = \cot^2 \sigma^1 , \qquad \tilde{g}_{11} = \sec^2 \sigma^1 , \qquad \tilde{g}_{22} = 4 . \tag{3.55}$$

The contraction yields,

$$\begin{aligned}\tilde{g}^{ab}\partial_a X^M \partial_b X^N \delta g^A_{MN} &= -\frac{2}{3} J s^A - \frac{2}{3} J s^A + J \tilde{g}^{11} \frac{(\partial_1 z)^2}{z^2} s^A + \frac{1}{3} J s^A \\ &= -J \sin^2 \sigma_1 s^A ,\end{aligned} \tag{3.56}$$

leading to the action variation

$$\begin{aligned}\delta S_{\text{M2}} &= T_{\text{M2}} \int d^3\sigma \left(2 \frac{\cos \sigma_1}{\sin^2 \sigma_1} \right) \frac{1}{2} (-J \sin^2 \sigma) s^A \mathbf{Y}^A \\ &= -J T_{\text{M2}} \int d^3\sigma \cos \sigma_1 s^A \mathbf{Y}^A .\end{aligned} \tag{3.57}$$

Using the AdS/CFT dictionary, the correlation function reads

$$\begin{aligned}\frac{\langle W[C] \mathcal{O}^A(y) \rangle}{\langle W[C] \rangle \sqrt{\mathcal{N}_{\mathcal{O}}}} &= -\frac{\delta S_{\text{DBI}}}{\delta s^A_0(y)} = J T_{\text{M2}} \int d^3\sigma \cos \sigma_1 G_\Delta(y; x, z) \mathbf{Y}^A \\ &= \frac{2^{\Delta + \frac{1}{4}} \sqrt{\pi (2\Delta + 1)}}{k \lambda^{\frac{1}{4}}} \frac{R^\Delta}{L^{2\Delta}} \mathbf{Y}^A ,\end{aligned} \tag{3.58}$$

where we have used (3.3), (3.15) and (3.48). We define the OPE coefficients $C_{\mathcal{O}^A}$ as

$$\frac{\langle W[C] \mathcal{O}^A(y) \rangle}{\langle W[C] \rangle \sqrt{\mathcal{N}_{\mathcal{O}}}} = C_{\mathcal{O}^A} \frac{R^\Delta}{L^{2\Delta}} \mathbf{Y}^A, \tag{3.59}$$

and obtain

$$C_{\mathcal{O}^A} = \frac{2^{\Delta + \frac{1}{4}} \sqrt{\pi (2\Delta + 1)}}{k \lambda^{\frac{1}{4}}} . \tag{3.60}$$

4 Field theory results from localization

4.1 The case of $\langle W[L] T_{\mu\nu} \rangle / \langle W[L] \rangle$

In this subsection, we compare our holographic computation of $\langle W[L] T_{\mu\nu} \rangle / \langle W[L] \rangle$ with field theory result. Symmetry fixes this correlation function to the form (2.26) up to a scaling function $h_W(k, N)$, which is known to be related to the Bremsstrahlung function $\mathcal{B}(k, N)$ by[8]

$$h_W(k, N) = \frac{\mathcal{B}(k, N)}{2}. \tag{4.1}$$

The Bremsstrahlung function for ABJM theory has been determined non-perturbatively in M-theory limit [61] based on [54], and its large N expansion (with finite k) takes the form

$$\mathcal{B}(k, N)\big|_{N \gg 1} = \frac{1}{2\pi} \sqrt{\frac{N}{2k}} - \frac{1}{2\pi k} \cot \frac{2\pi}{k} + \mathcal{O}(N^{-1/2}). \tag{4.2}$$

[8] This is first conjectured in [59] for general spacetime dimensions and proved for four-dimensional $\mathcal{N} = 2$ theories [77].

Consequently, the scaling function $h_W(N,k)$ in the M-theory limit is given by

$$h_W(k,N)|_{N\gg 1} = \frac{\sqrt{N}}{4\pi\sqrt{2k}} + \mathcal{O}(N^0). \tag{4.3}$$

This matches precisely with the holographic result (3.40).

4.2 The case of $\langle\mathcal{O}\rangle_{W[C]}$

In this subsection, we review the localization computation of the correlation function of a Wilson line and a CPO with conformal dimension $\Delta = 1$ [60] and compare it with the holographic result.[9] Following [60], we consider the ABJM theory on $\mathbb{R}^3$ with Euclidean signature. The half-BPS Wilson line is placed along the x^3-axis at $(x^1, x^2, x^3) = (0, 0, s)$ and the polarization vector α_I in the definition of the Wilson loop is chosen to be $\alpha_I = (1, 0, 0, 0)$.[10] Local operators are inserted at $(r\cos\tau, r\sin\tau, 0)$, and their polarization vectors are given by[11]

$$n_I = \frac{1}{\sqrt{2}}\left(e^{-\frac{\mathrm{i}}{2}\tau}, 0, e^{\frac{\mathrm{i}}{2}\tau}, 0\right), \qquad \bar{n}^I = \frac{1}{\sqrt{2}}\left(e^{\frac{\mathrm{i}}{2}\tau}, 0, -e^{-\frac{\mathrm{i}}{2}\tau}, 0\right). \tag{4.4}$$

The 1/3-BPS operator with $\Delta = 1$ in the translation-twisted frame along the circle is

$$\mathcal{O}(\tau) = \mathrm{Tr}(n_I(\tau)Y^I(\tau)\bar{n}^J(\tau)\bar{Y}_J(\tau)). \tag{4.5}$$

In this frame, the propagator d_{ij} takes the form

$$d_{12} = \frac{n(\tau_1)\cdot\bar{n}(\tau_2)}{|x(\tau_1) - x(\tau_2)|} = \frac{\mathrm{i}\sin\frac{\tau_1-\tau_2}{2}}{2r|\sin\frac{\tau_1-\tau_2}{2}|}, \tag{4.6}$$

leading to

$$d_{12}d_{21} = \frac{1}{4r^2}, \tag{4.7}$$

which is independent of τ_i. The two-point function of $\mathcal{O}$ is normalized as

$$\langle\mathcal{O}(\tau_1)\mathcal{O}(\tau_2)\rangle = \mathcal{N}_{\mathcal{O}} d_{12}d_{21}, \tag{4.8}$$

where $\mathcal{N}_{\mathcal{O}}$ is the normalization constant.

Localization calculation. For arbitrary finite N and k, the integrated two-point function is given by [78, 79]

$$\left\langle \int d\tau_1 \mathcal{O}(\tau_1) \int d\tau_2 \mathcal{O}(\tau_2) \right\rangle = -\frac{1}{16\pi^2 r^4}\frac{1}{Z}\frac{\partial^2 Z[m]}{\partial m^2}\bigg|_{m=0}, \tag{4.9}$$

[9] As we will show below, there is simple relation between the OPE coefficients of the Wilson line and the Wilson loop.

[10] The WL is put along x^3 direction, instead of the x^1 direction as in section 2. As a result, we should change the spinors (2.8) in the definition of the Wilson line accordingly for the computation. However, since we are only concerned about OPE coefficients, which is spacetime independent, the changes of the conventions do not affect the result.

[11] We have adapted to our notation.

where $Z[m]$ is the partition function of the mass-deformed ABJM theory where m is the mass-deformation parameter. Using localization it can be expressed as [12, 80–82]

$$Z[m] = \frac{1}{(N!)^2(2\pi)^{2N}} \int d\lambda_i d\mu_i \exp\left[\frac{ik}{4\pi}\left(\sum \lambda_i^2 - \sum \mu_i^2\right)\right] \tag{4.10}$$

$$\times \frac{\prod_{i\neq j}^N 2\sinh\frac{\lambda_i-\lambda_j}{2} \prod_{i\neq j}^N 2\sinh\frac{\mu_i-\mu_j}{2}}{\prod_{i,j} 2\cosh\frac{\lambda_i-\mu_j}{2} 2\cosh\left(\frac{\mu_j-\lambda_i}{2} - \pi m r\right)} . \tag{4.11}$$

Using Fermi gas approach, all order perturbatrive $1/N$ corrections to $Z[m]$ can be resumed.[12] The result is given in terms of the Airy function [53, 60]

$$Z^{\text{pert.}}[m] = e^A C^{-1/3} \text{Ai}[C^{-1/3}(N-B)] , \tag{4.12}$$

where A, B, C are functions of k, r, m. Here we do not need the explicit expressions for A and B. As for C, we have

$$C = \frac{2}{\pi^2 k(1+4m^2r^2)} . \tag{4.13}$$

Using the asymptotic expansion of the Airy function

$$\text{Ai}(x) \sim \exp\left(-\frac{2}{3}x^{3/2}\right) , \tag{4.14}$$

where an irrelevant factor $(1/x)^{1/4}/(2\sqrt{\pi})$ has been omitted, the large N limit of the integrated two-point function reads

$$\left\langle \int d\tau_1 \mathcal{O}(\tau_1) \int d\tau_2 \mathcal{O}(\tau_2) \right\rangle = -\frac{1}{16\pi^2 r^4} \frac{1}{Z[m]} \frac{\partial^2 Z[m]}{\partial m^2}\bigg|_{m=0} = \frac{\sqrt{2k}N^{3/2}}{12\pi r^2} . \tag{4.15}$$

Performing the τ-integrals on both sides of (4.8) yields

$$\left\langle \int d\tau_1 \mathcal{O}(\tau_1) \int d\tau_2 \mathcal{O}(\tau_2) \right\rangle = \frac{\pi^2 \mathcal{N}_{\mathcal{O}}}{r^2}. \tag{4.16}$$

Comparing (4.15) and (4.16) fixes the normalization

$$\mathcal{N}_{\mathcal{O}} = \frac{\sqrt{2k}N^{3/2}}{12\pi^3} . \tag{4.17}$$

The correlation function of the Wilson line $W[L]$ and $\mathcal{O}$ is given by

$$\frac{\langle W[L]\mathcal{O}(0)\rangle}{\langle W[L]\rangle} = \frac{\mathcal{B}}{r}, \tag{4.18}$$

where $\mathcal{B}$ is the Bremsstranhlung function (4.2). Including the normalization factor, we have

$$\frac{\langle W[L]\mathcal{O}(0)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L]\rangle} = \frac{\mathcal{B}}{\sqrt{\mathcal{N}_{\mathcal{O}}}r} . \tag{4.19}$$

The coefficient of $1/r$ is

$$\frac{\mathcal{B}}{\sqrt{\mathcal{N}_{\mathcal{O}}}} = \frac{\sqrt{3\pi}}{2^{3/4}k\lambda^{1/4}} , \tag{4.20}$$

which is the prediction from localization.

[12] We omit all instanton corrections.

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Comparison with holography. The holographic computation of the Wilson loop OPE coefficient for the CPO (4.5) gives

$$\mathcal{C}_{\mathcal{O}} = \frac{2^{5/4}\sqrt{3\pi}}{k\lambda^{1/4}}\,, \tag{4.21}$$

by setting $\Delta = 1$ in (3.60). At first glance, this does not match (4.20). However, note that the holographic correlation function for a normalized CPO of dimension Δ with the Wilson line takes the form

$$\frac{\langle W[L]\mathcal{O}\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L]\rangle} = \mathcal{C}_{\mathcal{O}}\mathbf{Y}^A\frac{1}{(2r)^{\Delta}}, \tag{4.22}$$

where r is the distance from the CPO to the line, and $\mathbf{Y}^A$ is the spherical harmonic factor.

For the Wilson line considered here, the polarization vector is $\alpha_I = (1,0,0,0)$, and the spherical harmonic evaluates to[13]

$$\mathbf{Y}^A = \bar{n}^I n_J \alpha_I \bar{\alpha}^J = 1/2\,. \tag{4.23}$$

Thus, the holographic prediction for $\Delta = 1$ is

$$\frac{\langle W[L]\mathcal{O}\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L]\rangle} = \frac{\mathcal{C}_{\mathcal{O}}\mathbf{Y}^A}{2r} = \frac{\sqrt{3\pi}}{2^{3/4}k\lambda^{1/4}r}\,, \tag{4.24}$$

which exactly matches the localization result (4.20).

The factor of 2^{Δ} in the denominator of (4.22) arises from the near-line limit of the correlation function. For a circular Wilson loop of radius R and a CPO at a generic point (y_1, y_2, y_3), the correlation function takes the form in (2.23) [44, 65]

$$\frac{\langle W[C]\mathcal{O}\rangle}{\langle W[C]\rangle\sqrt{\mathcal{N}_{\mathcal{O}}}} = \tilde{C}\frac{R^{\Delta}}{((h^2+\rho^2-R^2)^2+4R^2h^2)^{\Delta/2}}\,. \tag{4.25}$$

In the OPE limit $L = \sqrt{\rho^2+h^2} \gg R$, this reduces to

$$\frac{\langle W[C]\mathcal{O}\rangle}{\langle W[C]\rangle\sqrt{\mathcal{N}_{\mathcal{O}}}} = \tilde{C}\frac{R^{\Delta}}{L^{2\Delta}}\,. \tag{4.26}$$

Hence $\tilde{C}$ is identified with the OPE coefficient $\mathcal{C}_{\mathcal{O}}$.

For the Wilson line ($R \to \infty$), the correct limit is not obtained by naively sending $R \to \infty$, which is vanishing. Instead, we need to carefully send $r = \sqrt{(\rho-R)^2+h^2} \to 0$ with $(\rho - R)/h$ finite. The geometric meaning of this limit is that when the operator is very near to the circle, it cannot distinguish between the circle and the straight line. This limit yields

$$\frac{\langle W[C]\mathcal{O}\rangle}{\langle W[C]\rangle\sqrt{\mathcal{N}_{\mathcal{O}}}} \approx \frac{\mathcal{C}_{\mathcal{O}}}{(2d)^{\Delta}}. \tag{4.27}$$

This explains the factor of 2^{Δ} in (4.22).

[13]Recall that here the spherical harmonics is for $S^7/\mathbb{Z}_k$ with radius 1.

5 Conclusion

In this work, we have computed the correlation functions between a straight Wilson line (or a circular Wilson loop) and local BPS operators. Specifically, the local operator can be a stress energy tensor or a 1/3-BPS CPO in ABJM theory. By employing holographic methods, we derived explicit expressions for these one-point functions in the M-theory limit and compared them with predictions from supersymmetric localization. Our findings provide valuable strong coupling data for defect ABJM theory and establish a concrete basis for computing more general WL correlators holographically.

The matching between holographic and localization results underscores the power of combining complementary non-perturbative techniques in the study of defect CFTs. On the one hand, holography offers a geometric perspective, where WL correlators are mapped to the response of string or brane configurations to certain fluctuations of the bulk background fields. On the other hand, localization reduces the problem to finite dimensional matrix integrals, enabling exact computations even at finite N and k. There are many interesting directions one can pursue in the future.

Our work focused on half-BPS WLs in the fundamental representation of the supergroup $\mathrm{U}(N|N)$. A natural extension is to study WLs with less supersymmetries and/or in higher-rank representations, such as symmetric or antisymmetric representations. For example, one can consider the 1/6-BPS fermionic latitude WLs [83] with a string dual given in [84]. However, in many cases the string description can be quite complex, or probe string/brane solutions are still lacking [52]. For bosonic 1/6-BPS WLs in the fundamental representation, the holographic description involves smearing the string worldsheet over $\mathbb{CP}^1$ inside $\mathbb{CP}^3$ [48, 49]. For WLs in the antisymmetric representation, only the 1/6-BPS D6-brane solution is known, with half-BPS ones remaining elusive. For instance, the string dual of fermionic 1/6-BPS WLs [85, 86] in the fundamental representation of $\mathrm{U}(N|N)$ involves complicated mixed boundary conditions on the worldsheet [87]. Extending our methods to these cases would yield new insights into the dependence of the defect correlator and its bulk duals on representation and supersymmetry.[14] Similar half-BPS Wilson loops also exist in ABJ theory [88] with gauge group $\mathrm{U}(M) \times \mathrm{U}(N)$. The localization computations in [60] were actually performed in this general setting. However, because the holographic description of ABJ theory involves discrete torsion, finding dual gravitational descriptions for these half-BPS Wilson loops and their one-point functions remains challenging. We therefore leave this interesting problem for future work.

The computation of WL correlators lies at the intersection of multiple non-perturbative approaches, and further progress could be made by leveraging integrability and localization more broadly. While our work focused on specific BPS operators, a localization formula for generic CPO correlators would be highly desirable. Such a result could enable systematic studies of WL one-point functions beyond the stress energy tensor and 1/3-BPS CPOs with $\Delta = 1$. In the planar limit, integrability techniques could be applied to compute WL one-point functions for non-BPS operators. For WLs that correspond to integrable boundary states, the OPE coefficients might be computable using the worldsheet g-function approach [89, 90]. Preliminary steps in this direction have been taken at weak coupling [33], but a strong-coupling analysis remains an open challenge.

[14] In some cases, we would need to find the dual string/brane solutions first.

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While we focused on one-point functions, defect CFTs admit a richer class of observables, such as two-point functions of local operators in the presence of a WL, which encode defect operator spectra [55]. The computation of defect two-point functions are much more challenging as it depends on the conformal cross ratios in a non-trivial way. Recently, progress have been made in computing defect two-point functions in $\mathcal{N}=4$ SYM theory with defects of various co-dimensions by superconformal bootstrap [91, 92]. It is very interesting to see whether such techniques can be generalized to the ABJM theory.

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