Correlation functions in the TTbar deformed CFTs

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Outlook

- **Intro to TTar deformation**
- **Correlation functions in TT deformed theory （Plane, SUSY, Torus）**
- **Chaotic behavior in TT deformed theory**
- **Future Problems**

The $T\bar{T}$ operator

Consider the following bi-local operator in a 2d QFT.

$$T\bar{T}(z, z') = T_{zz}(z)T_{\bar{z}\bar{z}}(z') - T_{z\bar{z}}(z)T_{z\bar{z}}(z')$$

$z = x + it$

[Zamolodchikov]

The expectation value of this operator is given by the expectation values of the stress tensor itself and is a constant.

$$\langle T\bar{T}\rangle = \langle T_{zz}\rangle\langle T_{\bar{z}\bar{z}}\rangle - \langle T_{z\bar{z}}\rangle^2$$

This is true very generally in a reasonably well behaved 2d QFT which has a local conserved stress tensor.

The operator can also be defined (upto total derivatives) at coincident points.

$$T\bar{T} \equiv T_{zz}T_{\bar{z}\bar{z}} - T_{z\bar{z}}^2$$

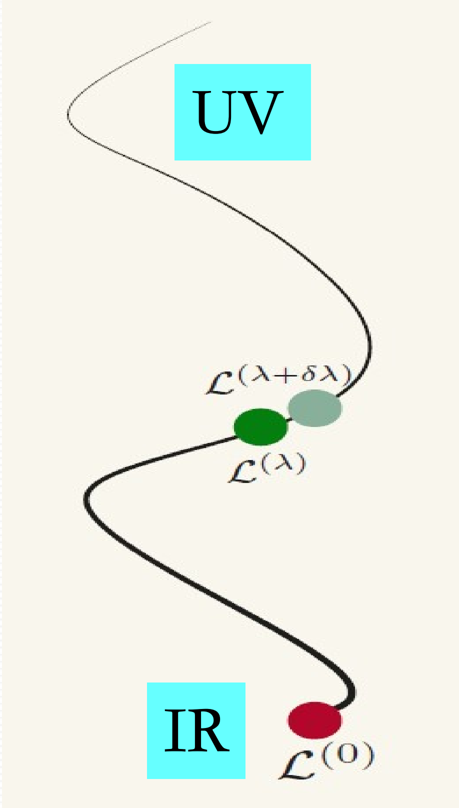
Higher Dimensional Deformation

$$\det T = \frac{1}{2}\epsilon_{ik}\epsilon_{jl}T^{ij}T^{kl}$$

Deformation of TTbar

$$\mathcal{L}^{(\lambda+\delta\lambda)} = \mathcal{L}^{(\lambda)} + \delta\lambda\, T\bar{T}$$

$$\frac{dS(\lambda)}{d\lambda} = \int d^2x\, T\bar{T}(x).$$



- The spectrum of the deformed theory can be solved exactly and non-perturbatively.

[Smirnov-Zamolodchikov; Cavaglia-Negro-Szecsenyi-Tateo]

- Deforming an integrable QFT by this operator preserves integrability.

[F. A. Smirnov and A. B. Zamolodchikov,16]

- Deforming by $T\bar{T}$ = coupling the theory to Jackiw-Teitelbohm gravity

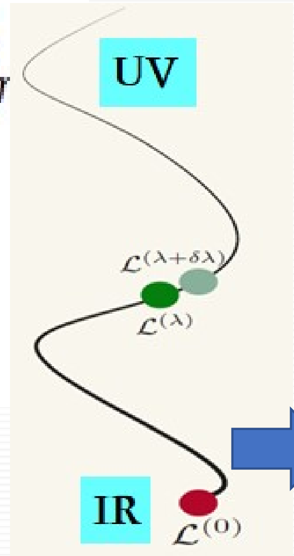
[Dubovsky-Gorbenko-…]

- **Correlation functions in deep UV of the deformed theory given by difussion equation** [Cardy, 19]

$$\langle \prod_n \Phi_n(x_n)\rangle_\lambda = \int \prod_n G(x_n - y_n;\tilde{\lambda})\langle \prod_n \Phi_n(y_n)\rangle_0 \prod_n d^2 y_n$$

where

$$G(x-y;\tilde{\lambda}) = (4\pi\tilde{\lambda})^{-1} e^{-(x-y)^2/4\tilde{\lambda}}$$



- **By canonical transformation in phase space, Flow equation of correlation functions** [Jorrit Kruthoff, Onkar Parrikar,20]

Correlation functions in deformed theory

Zero-pt Correlation function:[Aharony-Shouvik-Giveon-Jiang-Kutasov,18]; [Shouvik-Jiang,18][Cardy,19]···.

$$\lambda \int d^2 z \langle T\bar{T}(z)\phi_1(z_1)\ldots\phi_n(z_n)\rangle$$

N<5 Point, S. He, Hongfei Shu [1907.12603]

The deformation of correlation function

$$\langle \mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\cdots\mathcal{O}_n(z_n,\bar{z}_n)\rangle_\lambda = \lambda \int d^2z \langle T\bar{T}(z,\bar{z})\mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\cdots\mathcal{O}_n(z_n,\bar{z}_n)\rangle$$

Energy-Momentum conservation

$$\langle \mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\cdots\mathcal{O}_n(z_n,\bar{z}_n)\rangle_\lambda = \lambda \int d^2z \Big(\sum_{i=1}^{n}\big(\frac{h_i}{(z-z_i)^2}+\frac{\partial_{z_i}}{z-z_i}\big)\Big)\Big(\sum_{i=1}^{n}\big(\frac{\bar{h}_i}{(\bar{z}-\bar{z}_i)^2}+\frac{\partial_{\bar{z}_i}}{\bar{z}-\bar{z}_i}\big)\Big) \times \langle \mathcal{O}_1(z_1,\bar{z}_1)\mathcal{O}_2(z_2,\bar{z}_2)\cdots\mathcal{O}_n(z_n,\bar{z}_n)\rangle,$$

Ward Identity

The deformed Two-Point Function(Step 1)

$$\lim_{\ell\to 0}\langle T(z+\ell)\bar{T}(\bar{z}-\ell)\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle$$

$$=\lim_{\ell\to 0}\Big(\sum_{i=1}^{2}\big(\frac{\bar{h}}{(\bar{z}-\bar{z}_i-\ell)^2}+\frac{\partial_{\bar{z}_i}}{\bar{z}-\bar{z}_i-\ell}\big)\Big)\Big(\sum_{i=1}^{2}\big(\frac{h}{(z-z_i+\ell)^2}+\frac{\partial_{z_i}}{z-z_i+\ell}\big)\Big)\langle\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle$$

Regularization （Step 2）

Log term associated with boundary term of Non local deformation(TTar). Nonlocal divergence given by Peskin ‘s QFT.

$$\langle T(z)\bar{T}(z)\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle_\lambda=\lambda h\bar{h}z_{12}^2\bar{z}_{12}^2\mathcal{I}_2(z_1,z_2)\langle\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle$$

$$=\lambda h\bar{h}\frac{8\pi}{|z_{12}|^2}\Big(\frac{4}{\epsilon}+2\log|z_{12}|^2+2\log\pi+2\gamma-5\Big)\langle\mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\rangle,$$

[P. Kraus, J. Liu and D. Marolf，18; Cardy, 19]

Four-Point in CFTs

$$\langle \mathcal{O}^\dagger(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\mathcal{O}^\dagger(z_3,\bar{z}_3)\mathcal{O}(z_4,\bar{z}_4)\rangle = \frac{G(\eta,\bar{\eta})}{z_{13}^{2h} z_{24}^{2h} \bar{z}_{13}^{2\bar{h}} \bar{z}_{24}^{2\bar{h}}},$$

with the cross ratios

$$\eta = \frac{z_{12}z_{34}}{z_{13}z_{24}}, \quad \bar{\eta} = \frac{\bar{z}_{12}\bar{z}_{34}}{\bar{z}_{13}\bar{z}_{24}}.$$

Four-Point in Deformed-CFTs

$$
\begin{aligned}
&\langle \mathcal{O}(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\mathcal{O}^\dagger(z_3,\bar{z}_3)\mathcal{O}(z_4,\bar{z}_4)\rangle_\lambda \\
=&\lambda\Big\{h\bar{h}z_{13}^2\bar{z}_{13}^2\mathcal{I}_{2222}(z_1,z_3,\bar{z}_1,\bar{z}_3)+h\bar{h}z_{24}^2\bar{z}_{13}^2\mathcal{I}_{2222}(z_2,z_4,\bar{z}_1,\bar{z}_3) \\
&+h\bar{h}z_{13}^2\bar{z}_{24}^2\mathcal{I}_{2222}(z_1,z_3,\bar{z}_2,\bar{z}_4)+h\bar{h}z_{24}^2\bar{z}_{24}^2\mathcal{I}_{2222}(z_2,z_4,\bar{z}_2,\bar{z}_4) \\
&+\big(\bar{z}_{13}^2\mathcal{I}_{111122}(z_1,z_2,z_3,z_4,\bar{z}_1,\bar{z}_3)+\bar{z}_{24}^2\mathcal{I}_{111122}(z_1,z_2,z_3,z_4,\bar{z}_2,\bar{z}_4)\big)\bar{h}z_{23}z_{14}\frac{\eta\partial_\eta G(\eta,\bar{\eta})}{G(\eta,\bar{\eta})} \\
&+\big(z_{13}^2\mathcal{I}_{221111}(z_1,z_3,\bar{z}_1,\bar{z}_3,\bar{z}_2,\bar{z}_4)+z_{24}^2\mathcal{I}_{221111}(z_2,z_4,\bar{z}_2,\bar{z}_4,\bar{z}_1,\bar{z}_3)\big)h\bar{z}_{23}\bar{z}_{14}\frac{\bar{\eta}\partial_{\bar{\eta}} G(\eta,\bar{\eta})}{G(\eta,\bar{\eta})} \\
&+z_{23}z_{14}\bar{z}_{23}\bar{z}_{14}\mathcal{I}_{11111111}(z_1,z_2z_3,z_4,\bar{z}_1,\bar{z}_3,\bar{z}_2,\bar{z}_4)\eta\bar{\eta}\frac{\partial_\eta\partial_{\bar{\eta}} G(\eta,\bar{\eta})}{G(\eta,\bar{\eta})}\Big\} \\
&\langle \mathcal{O}^\dagger(z_1,\bar{z}_1)\mathcal{O}(z_2,\bar{z}_2)\mathcal{O}^\dagger(z_3,\bar{z}_3)\mathcal{O}(z_4,\bar{z}_4)\rangle.
\end{aligned}
$$

N-point Function, SYSU $\cdots$

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

Correlation functions in deformed CFTs with SUSY (1,1) (2,2)

H. Jiang, A. Sfondrini and G. Tartaglino-Mazzucchelli,19; （0，2）
C. K. Chang, C. Ferko, S. Sethi, A. Sfondrini and G. Tartaglino-Mazzucchelli, 19;（1，1）
E. A. Coleman, J. Aguilera-Damia, D. Z. Freedman and R. M. Soni,19;（2，2）
H. Jiang and G. Tartaglino-Mazzucchelli, 19（JT）

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

SUSY transformation

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coordinates on superspace $Z = (z, \theta)$ $\bar{Z} = (\bar{z}, \bar{\theta})$

$$D = \partial_\theta + \theta \partial_z, \quad D^2 = \partial_z.$$

$$\oint dZ \equiv \frac{1}{2\pi i} \oint dz \int d\theta.$$

supercoordinates transformations $\delta_E \Phi(Z, \bar{Z}) = [J_E, \Phi(Z, \bar{Z})] = \oint dZ' E(Z') J(Z') \Phi(Z, \bar{Z})$

A superfield $\Phi(Z, \bar{Z})$

$$\delta_E \Phi(Z, \bar{Z}) = E(Z) \partial_z \Phi(Z, \bar{Z}) + \frac{1}{2} D E(Z) D \Phi(Z, \bar{Z}) + \Delta \partial_z E(Z) \Phi(Z, \bar{Z}),$$

Ward ID SUSY

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

OPE

$$J(Z_1)\Phi(Z_2) = \frac{\theta_{12}}{Z_{12}}\partial_{z_2}\Phi(Z_2,\bar{Z}_2) + \frac{1}{2}\frac{1}{Z_{12}}D\Phi(Z_2,\bar{Z}_2) + \Delta\frac{\theta_{12}}{Z_{12}^2}\Phi(Z_2,\bar{Z}_2).$$

Ward ID SUSY：

$$J(Z) = \Theta(z) + \theta T(z)$$

$$\langle J(Z_0)\Phi_1(Z_1,\bar{Z}_1)\ldots\Phi_n(Z_n,\bar{Z}_n)\rangle$$
$$= \sum_{i=1}^{n}\left(\frac{\theta_{0i}}{Z_{0i}}\partial_{z_i} + \frac{1}{2Z_{0i}}D_i + \Delta_i\frac{\theta_{0i}}{Z_{0i}^2}\right)\langle\Phi_1(Z_1,\bar{Z}_1)\ldots\Phi_n(Z_n,\bar{Z}_n)\rangle.$$

Correlation function with （1,1） SUSY

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

2-Pt

$$\langle \Phi_1(Z_1,\bar{Z}_1)\Phi_2(Z_2,\bar{Z}_2)\rangle = c_{12}\frac{1}{Z_{12}^{2\Delta}\bar{Z}_{12}^{2\bar{\Delta}}}, \quad \Delta \equiv \Delta_1 = \Delta_2, \quad \bar{\Delta} \equiv \bar{\Delta}_1 = \bar{\Delta}_2$$

3-Pt

$$\langle \Phi_1(Z_1,\bar{Z}_1)\Phi_2(Z_2,\bar{Z}_2)\Phi_3(Z_3,\bar{Z}_3)\rangle = \Big(\prod_{i<j=1}^{3}\frac{1}{Z_{ij}^{\Delta_{ij}}\bar{Z}_{ij}^{\bar{\Delta}_{ij}}}\Big)(c_{123}+c'_{123}\theta_{123}\bar{\theta}_{123}),$$

$$c_{123}+c'_{123}\theta_{123}\bar{\theta}_{123} = c_{123}e^{c'_{123}\theta_{123}\bar{\theta}_{123}/c_{123}}. \quad \theta_{ijk} = \frac{1}{\sqrt{Z_{ij}Z_{jk}Z_{kl}}}(\theta_i Z_{jk}+\theta_j Z_{ki}+\theta_k Z_{ij}+\theta_i\theta_j\theta_k),$$

Correlation function with （1,1） SUSY

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

n-PT

$$\langle \Phi_1(Z_1, \bar{Z}_1)...\Phi_n(Z_n, \bar{Z}_n)\rangle = \Big(\prod_{i<j=1}^{n} \frac{1}{Z_{ij}^{\Delta_{ij}} \bar{Z}_{ij}^{\bar{\Delta}_{ij}}} \Big) f(w_i, \bar{w}_i, U_j, \bar{U}_j)$$

$$w_j \equiv \theta_{12j},\ j = 3, ..., n, \quad U_k \equiv Z_{123k},\ k = 4, ..., n,$$

$$Z_{ijkl} = \frac{Z_{ij} Z_{kl}}{Z_{li} Z_{jk}}.$$

First order deformation **2n-5** **OSP(2|1)**

$$-\lambda \int d^2 z \int d\theta d\bar{\theta} \langle J(Z) \bar{J}(\bar{Z}) \Phi(Z_1, \bar{Z}_1)...\Phi(Z_n, \bar{Z}_n)\rangle.$$

Deformed Correlation function with （1,1）SUSY

2-Pt

S. He, Jia-Rui Sun, Yuan Sun [1912.11461]

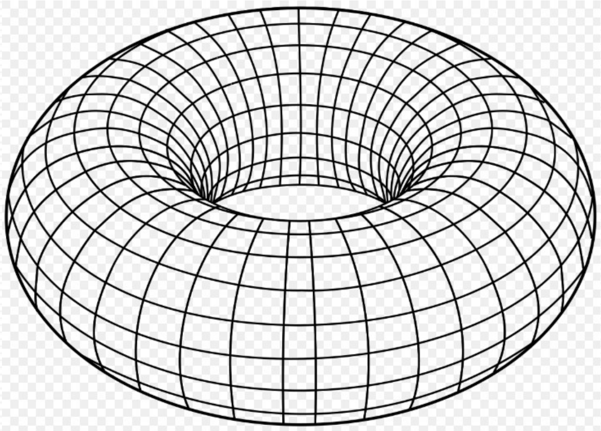
$$\int d^2z d\theta d\bar{\theta} \langle J(Z)\bar{J}(\bar{Z})\Phi_1(Z_1,\bar{Z}_1)\Phi_n(Z_2,\bar{Z}_2)\rangle / \langle \Phi_1(Z_1,\bar{Z}_1)\Phi_n(Z_2,\bar{Z}_2)\rangle$$
$$=\Delta\bar{\Delta}\int d^2z d\theta d\bar{\theta}\Big[\Big(-\frac{2}{Z_{12}}\Big(\frac{\theta_{01}}{z_{01}}-\frac{\theta_{02}}{z_{02}}\Big)-\frac{\theta_{12}}{Z_{12}}\Big(\frac{1}{Z_{01}}+\frac{1}{Z_{02}}\Big)+\Big(\frac{\theta_{01}}{z_{01}^2}+\frac{\theta_{02}}{z_{02}^2}\Big)\Big)$$
$$\times\Big(-\frac{2}{\bar{Z}_{12}}\Big(\frac{\bar{\theta}_{01}}{\bar{z}_{01}}-\frac{\bar{\theta}_{02}}{\bar{z}_{02}}\Big)-\frac{\bar{\theta}_{12}}{\bar{Z}_{12}}\Big(\frac{1}{\bar{Z}_{01}}+\frac{1}{\bar{Z}_{02}}\Big)+\Big(\frac{\bar{\theta}_{01}}{\bar{z}_{01}^2}+\frac{\bar{\theta}_{02}}{\bar{z}_{02}^2}\Big)\Big)\Big].$$

First order deformation

$$\frac{1}{\langle \Phi_1(Z_1,\bar{Z}_1)\Phi_2(Z_2,\bar{Z}_2)\rangle}\int d^2z d\theta d\bar{\theta}\langle J(Z)\bar{J}(\bar{Z})\Phi_1(Z_1,\bar{Z}_1)\Phi_2(Z_2,\bar{Z}_2)\rangle$$
$$=-\frac{4\pi\Delta^2}{Z_{12}\bar{Z}_{12}}\Big(-\frac{4}{\epsilon}+2\ln|z_{12}|^2+2\gamma+2\ln\pi-2\Big).$$

Similar structure as bosonic theory

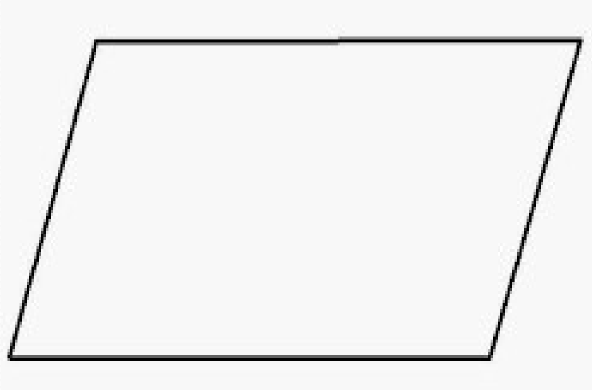
Correlation functions in deformed CFTs on Torus



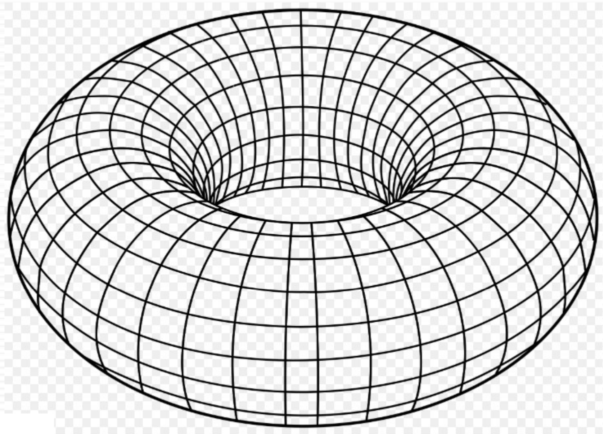
S. He, Yuan Sun [2004.07486]

Correlation function on Torus

S. He, Yuan Sun [2004.07486]



$$z = e^{2\pi i w}$$

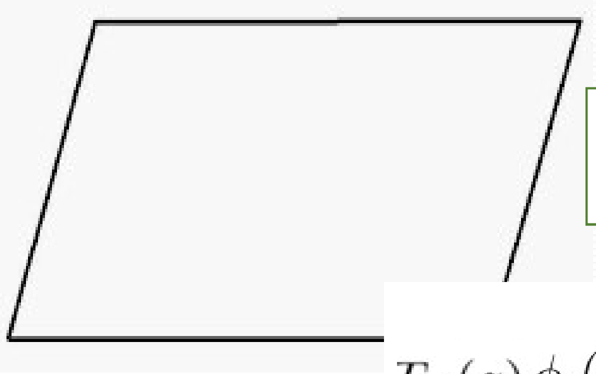


$$z^2 T_{pl}(z) = \frac{1}{(2\pi i)^2} T(w) + \frac{c}{24}$$

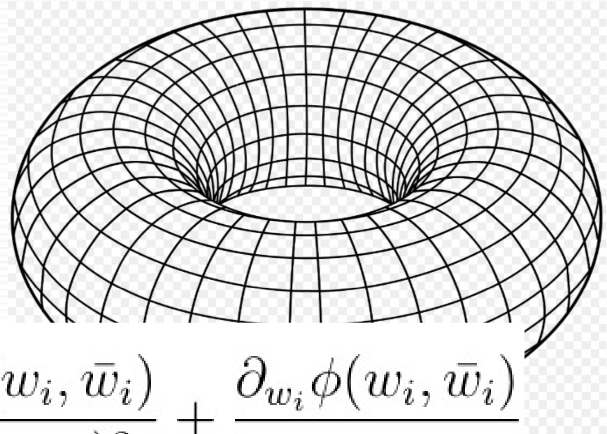
$$T_{pl}(z)\,\phi_i(w_i, \bar{w}_i) \sim \frac{h_i \phi_i(w_i, \bar{w}_i)}{(w - w_i)^2} + \frac{\partial_{w_i} \phi(w_i, \bar{w}_i)}{w - w_i}$$

Correlation function on Torus

S. He, Yuan Sun [2004.07486]



$$z = e^{2\pi i w}$$



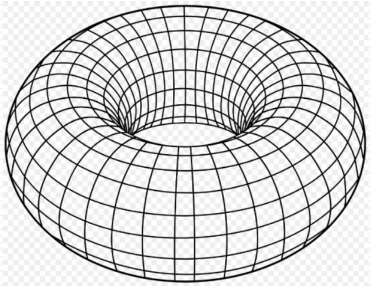
$$T_{pl}(z)\,\phi_i(w_i,\bar{w}_i) \sim \frac{h_i\phi_i(w_i,\bar{w}_i)}{(w-w_i)^2} + \frac{\partial_{w_i}\phi(w_i,\bar{w}_i)}{w-w_i}$$

$$\begin{aligned}&\langle T(w)X\rangle - \langle T\rangle\langle X\rangle \\ =&\sum_i \Big(h_i(P(w-w_i)+2\eta_1) + (\zeta(w-w_i)+2\eta_1 w_i)\partial_{w_i}\Big)\langle X\rangle + 2\pi i\partial_\tau\langle X\rangle,\end{aligned}$$

Weierstrass P-function and zeta function $P(w)\sim 1/w^2, \zeta(w)\sim 1/w$

Ward Identity on Torus

S. He, Yuan Sun [2004.07486]



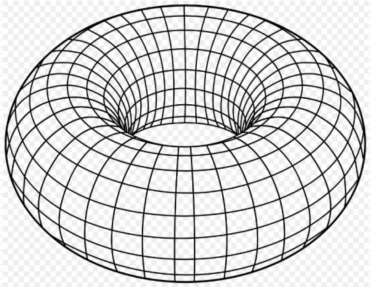
$$
\begin{aligned}
&\langle T(w)\bar{T}(\bar{v})X\rangle \\
=&2\pi i\partial_\tau\langle\bar{T}(\bar{v})X\rangle + 2\pi i(\partial_\tau \ln Z)\langle\bar{T}(\bar{v})X\rangle \\
&+\sum_i h_i\Big(-\zeta'(w_i-w)+2\eta_1\Big)\langle\bar{T}(\bar{v})X\rangle \\
&+\sum_i\Big(-\zeta(w_i-w)+2\eta_1 w_i-2\eta_1 w-\pi i\Big)\partial_{w_i}\langle\bar{T}(\bar{v})X\rangle
\end{aligned}
$$

Regularlization

$$
\lambda\int_{T^2-\sum_i D(w_i)} d^2v\langle T(v)\bar{T}(\bar{v})X\rangle,
$$

First order deformation on Torus

S. He, Yuan Sun [2004.07486]



$$Z' = \int D\phi e^{-S+\lambda \int d^2 z T\bar{T}(z)}$$
$$= Z(1 + \lambda \int d^2 z \langle T\bar{T}\rangle(z)) + \frac{1}{2}\lambda^2 \int \int d^2 u_1 d^2 u_2 \langle T\bar{T}(u_1) T\bar{T}(u_2)\rangle + \dots).$$

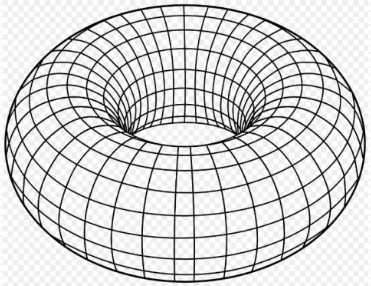
$$\lambda Z \int d^2 z \langle T\bar{T}\rangle(z) = \lambda (2\pi)^2 \tau_2 \partial_\tau \partial_{\bar{\tau}} Z.$$

$$Z = \mathrm{tr}(q^{L_0 - c/24} \bar{q}^{\bar{L}_0 - c/24}), \quad q = e^{2\pi i \tau}$$

- **Consistent with 1806.07426 [hep-th]**
- **Correlation function of KdV Operator is Modular invariant.**
- **Correlation function of Generic Operator may not be Modular invariant.**

First order deformation on Torus

S. He, Yuan Sun [2004.07486]



$$\mathrm{tr}(T(w)[T(u_1)...T(u_n)\bar{T}(v_1)...\bar{T}(v_m)]Xq^{L_0-c/12})$$

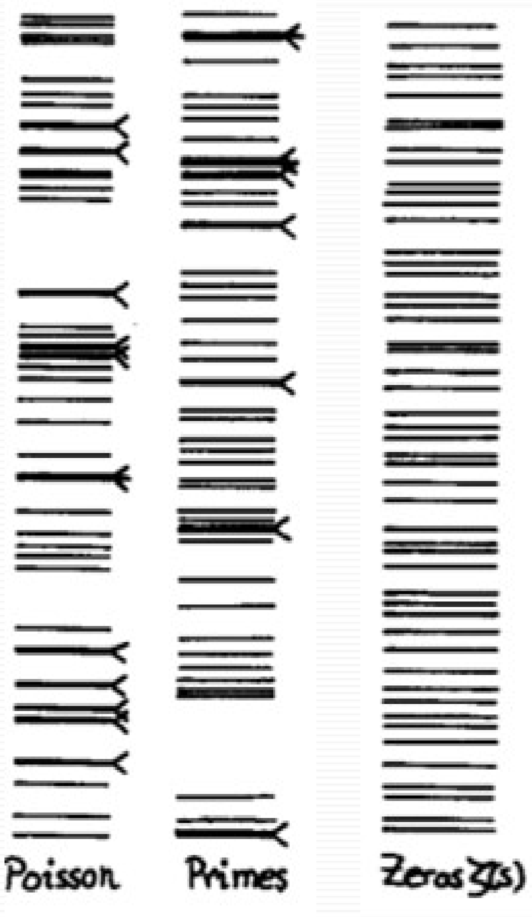
- **Can be computable but very complicated.**
- **New recursion relation found!**
- **Algorithm is offered!**

$$\langle T\rangle_\lambda - \langle T\rangle = \lambda\Big(- \frac{(2\pi i)^3 \tau_2 \partial_\tau^2 \partial_{\bar\tau} Z}{Z} + (2\pi)^3 \frac{\partial_\tau \partial_{\bar\tau} Z}{Z} - (2\pi i)\frac{\partial_\tau Z}{Z}\frac{(2\pi)^2 \tau_2 \partial_\tau \partial_{\bar\tau} Z}{Z}\Big)$$

where $\langle T\rangle = 2\pi i \partial_\tau \ln Z.$

Chaotic Behavior of deformed CFTs

A Chaotic spectrum



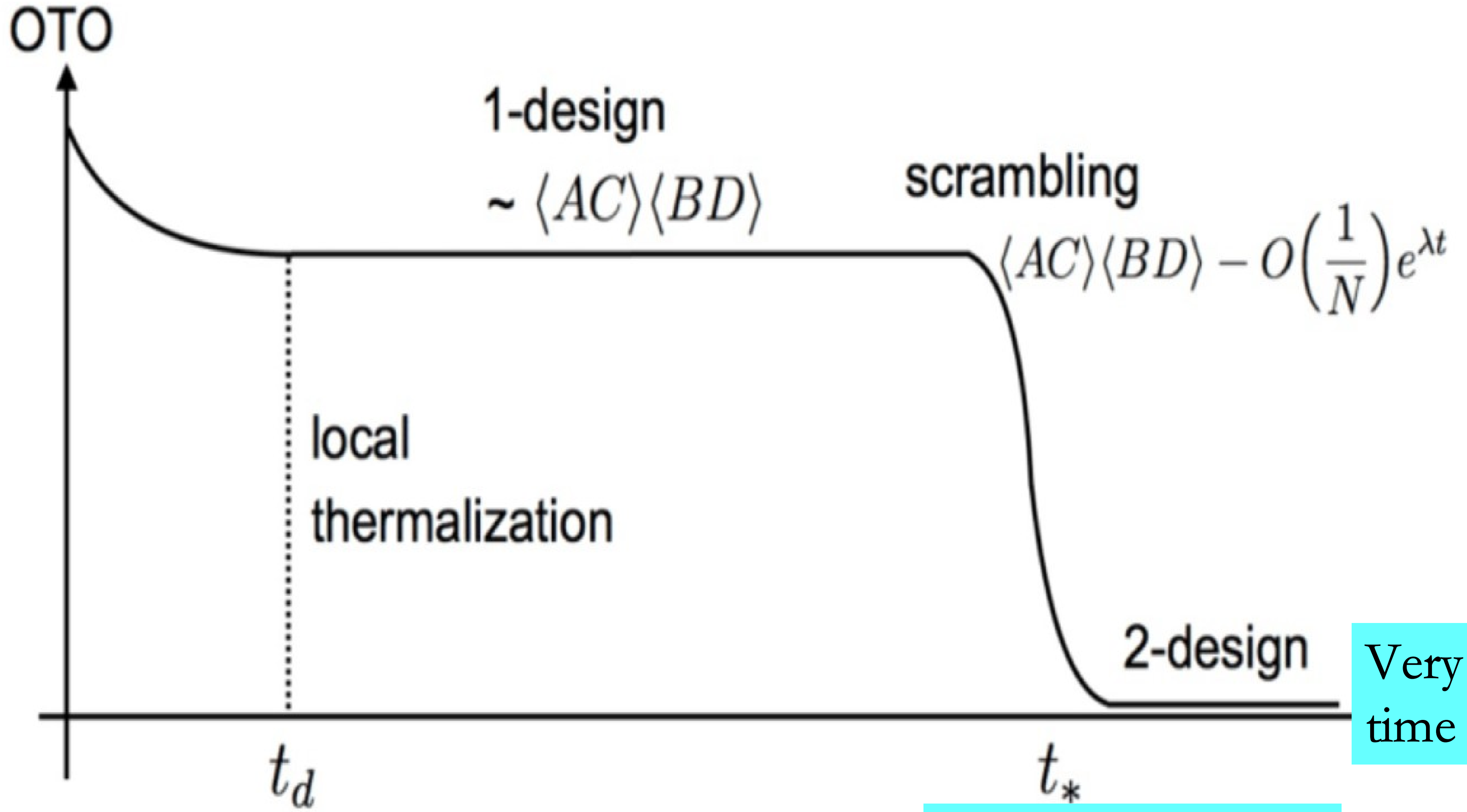
- Spectrum V.S. Quantum Chaos:
- Various spectra, the chaotic ones exhibit level repulsion and spectral rigidity.
- Their unfolded spectral distribution is a Wigner distribution P(s)~s^(beta)exp(-s^2).
- These properties are shared by various ensembles of random matrices.

OTOC can diagnose the chaotic behavior of Many body system.

- **In Chaotic system, the late time behavior of physical quantities are very sensitive to the early time input.**
- **There are many quantities to capture the behavior:**

  Lyapnov parameter, scrambling time scale and Rulle resonance. [A. Larkin and Y. Ovchinnikov,1969],[A. Kitaev,15]
- **For (non) integrable models do not show any chaotic signals, e.g OTOC (2^{nd} REE).** [E. Perlmutter,16],[Y. Gu and X. L. Qi,16].[S. He, Feng-Li Lin, J.J. Zhang, JHEP 1708 (2017) 126; JHEP 1712 (2017) 073].
- **One can also look at the spectrum form factor to test the time evolution behavior.**
- **By empirism, Holographic CFTs should have chaotic signals.**

  [E. Perlmutter,16],[J. L. Karczmarek, J. M. Maldacena and A.Strominger,16],[J. M. Maldacena, D. Stanford,16].



Very late time signal

Thermal dissipation time

Scrambling time

OTOC in TT-deformed CFTs

$$\frac{\langle W(t)VW(t)V\rangle_\beta}{\langle W(t)W(t)\rangle_\beta\langle VV\rangle_\beta}$$

Put the excitations on the thermal CFTs (Cylinder)

$$\frac{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)\rangle_\beta\langle V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}$$
$$\times\Big(1-\lambda(\frac{2\pi}{\beta})^2\int d^2z_b|z_b|^2\frac{\langle\big(T(z_b)-\frac{c}{24z^2}\big)\big(\bar{T}(\bar{z}_b)-\frac{c}{24\bar{z}^2}\big)W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)\rangle}{\langle W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)\rangle}$$
$$-\lambda(\frac{2\pi}{\beta})^2\int d^2z_c|z_c|^2\frac{\langle\big(T(z_c)-\frac{c}{24z^2}\big)\big(\bar{T}(\bar{z}_c)-\frac{c}{24\bar{z}^2}\big)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}{\langle V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}$$
$$+\lambda(\frac{2\pi}{\beta})^2\int d^2z_a|z_a|^2\frac{\langle\big(T(z_a)-\frac{c}{24z^2}\big)\big(\bar{T}(\bar{z}_a)-\frac{c}{24\bar{z}^2}\big)W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}{\langle W(z_1,\bar{z}_1)W(z_2,\bar{z}_2)V(z_3,\bar{z}_3)V(z_4,\bar{z}_4)\rangle}+\mathcal{O}(\lambda^2)\Big)$$

Late time of OTOC

S. He, Hongfei Shu [1907.12603]

$$\frac{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)\rangle_\beta\langle V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}$$
$$\xrightarrow{T\bar{T}} \frac{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}{\langle W(w_1,\bar{w}_1)W(w_2,\bar{w}_2)\rangle_\beta\langle V(w_3,\bar{w}_3)V(w_4,\bar{w}_4)\rangle_\beta}\left\{1-\lambda C_1(x)+\lambda C_2(x)e^{-\frac{2\pi}{\beta}t}+\cdots\right\},$$

The choices of the sign of λ do not affect the late time behavior exp[-2πβt] in above equation.

D. J. Gross, J. Kruthoff, A. Rolph and E. Shaghoulian,19

Summary

- Correlation functions in deformed theory on the plane and torus with/without SUSY.
- Entanglement, OTOC in deformed theory.
- The situation of TJ deformation is parallel to the TT deformed theory.

Future Problems

- Higher order correlation functions in deformed theory. [Yuan, Yu-Xuan]
- Entanglement entropy, Renyi Entropy, etc. in deformed CFTs
- Entanglement of Purification (EOP) of Local excited states. (6-pt)
- Complexity, Psuedo Entanglement entropy of deformed theory,
- Correlation function in Deep UV [Cardy,19; 2006.03054](Non-local)
- Modular structure of higher point correlation functions on torus (Sub class).
- 2D Bosonization of TTbar.

Thanks for your attention!