Anomalous transports in magnetized plasma at strong & weak coupling

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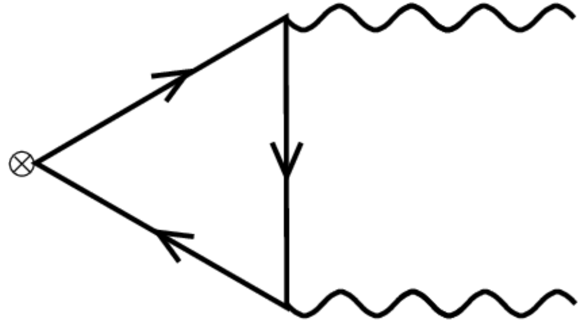
based on Bu, SL, EPJC(2020)
SL, Yang, PRD (2020), JHEP(2021)

Outline

- Anomalies and transports in hydrodynamics
- Experimental realizations of chiral magnetic/vortical effect
- Dual effect: magneto-vortical effect
- Magneto-vortical effect from holography
- Magneto-vortical effect from kinetic theory
- Conclusion and Outlook

(covariant) Chiral anomalies

$$J^\mu = \bar{\psi}\gamma^\mu\psi \qquad J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$$



classically $\partial_\mu J_5^\mu = 0$

radiative correction $\partial_\mu J_5^\mu = -\frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + (F \to F_5)$

axial charge approximately conserved for weak external fields

Hydrodynamics w/o anomaly

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad \partial_\mu J^\mu = 0$$

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu}$$

$$J^\mu = \rho u^\mu + \nu^\mu$$

entropy growth

$$\partial_\mu s^\mu \geq 0$$

$$s^\mu = s u^\mu - \frac{\mu}{T} \nu^\mu \qquad \nu^\mu = \sigma \left(E^\mu - T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right)$$

normal transports dissipative, constrained by entropy

Hydrodynamics with anomaly

$$\partial_\mu T^{\mu\nu} = F_5^{\nu\lambda} J_{5\lambda} \qquad \partial_\mu J_5^\mu = C E_5^\mu B_{5\mu}$$

think of a world with axial charge and axial fields only

$$J_5^\mu = \rho_5 u^\mu + \nu^\mu$$

entropy growth

$$\partial_\mu s^\mu \geq 0$$

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

$$\nu^\mu = \sigma\left(E_5^\mu - T P^{\mu\nu}\partial_\nu\left(\frac{\mu_5}{T}\right)\right) + (C\mu_5^2 + \# T^2)\omega^\mu + C\mu_5 B_5^\mu$$

Anomlous transports non-dissipative, almost fully determined by entropy

Son, Surowka, PRL(2009)
Neiman, Oz, JHEP(2011)

Known anomalous transports

Chiral Magnetic/Separation Effect (CME/CSE)

$$\mathbf{J} = C\mu_5 e\mathbf{B} \quad \mathbf{J}_5 = C\mu e\mathbf{B}$$

Vilenken, PRD 1980
Metlitski, Zhitnitsky, PRD 2005
Kharzeev, McLerran, Warringa, NPA 2008

Chiral Vortical Effect (CVE)

$$\mathbf{J} = C\mu_5\mu\boldsymbol{\omega} \quad \mathbf{J}_5 = C\left(\mu^2 + \mu_5^2 + \frac{\pi^2T^2}{3}\right)\boldsymbol{\omega}$$

Vilenken, PRD 1979
Erdmenger et al, JHEP 2009
Banerjee et al, JHEP 2011

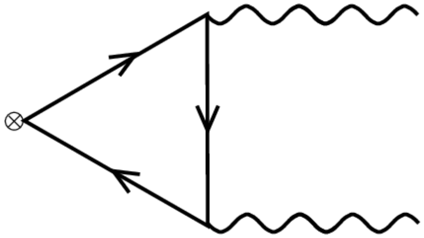
T^2 and gravitational anomaly

$$\mathbf{J}_5 = C\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)\boldsymbol{\omega}$$

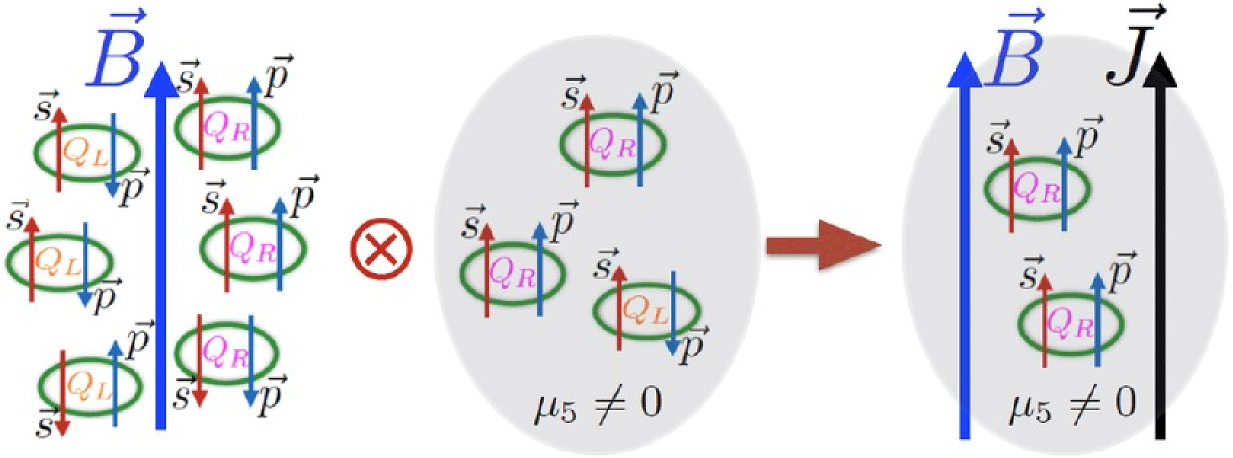
$$\partial_\mu J_5^\mu = -\frac{1}{384\pi^2}\varepsilon^{\mu\nu\rho\sigma} R^\alpha_{\beta\mu\nu} R^\beta_{\alpha\rho\sigma}$$

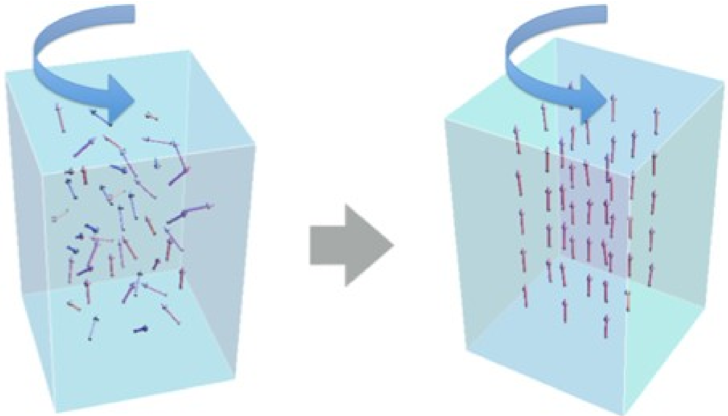
$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN} + \epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)\right]$$

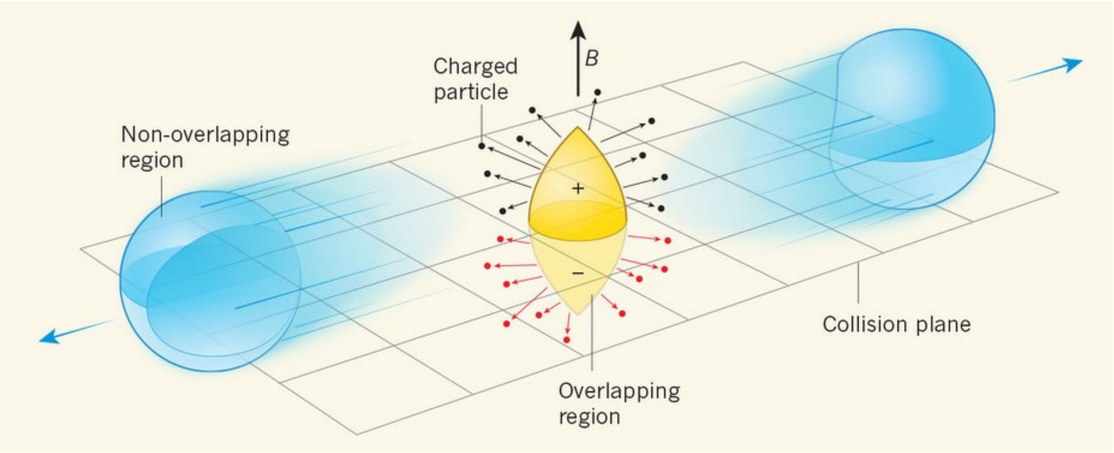
Landsteiner et al, PRL 2011, JHEP 2011

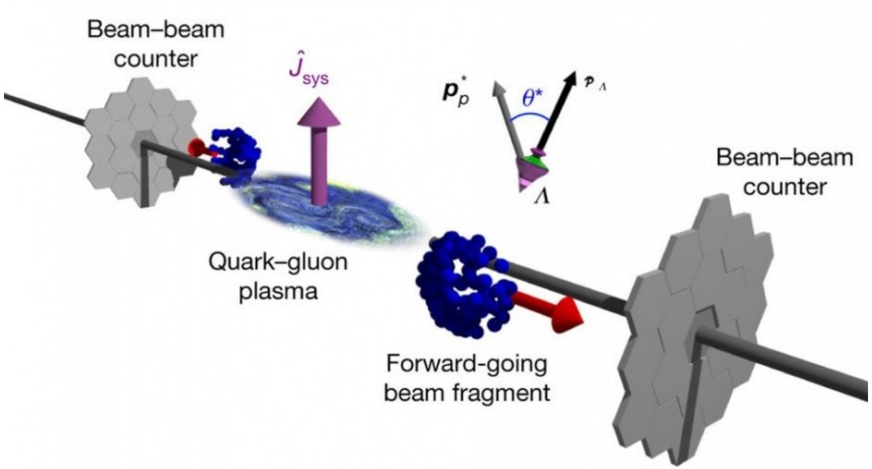


Spin alignment picture

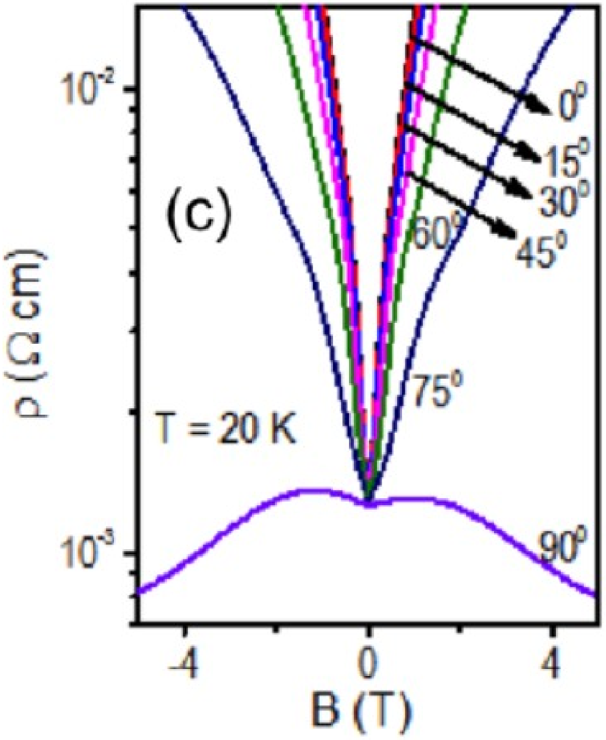






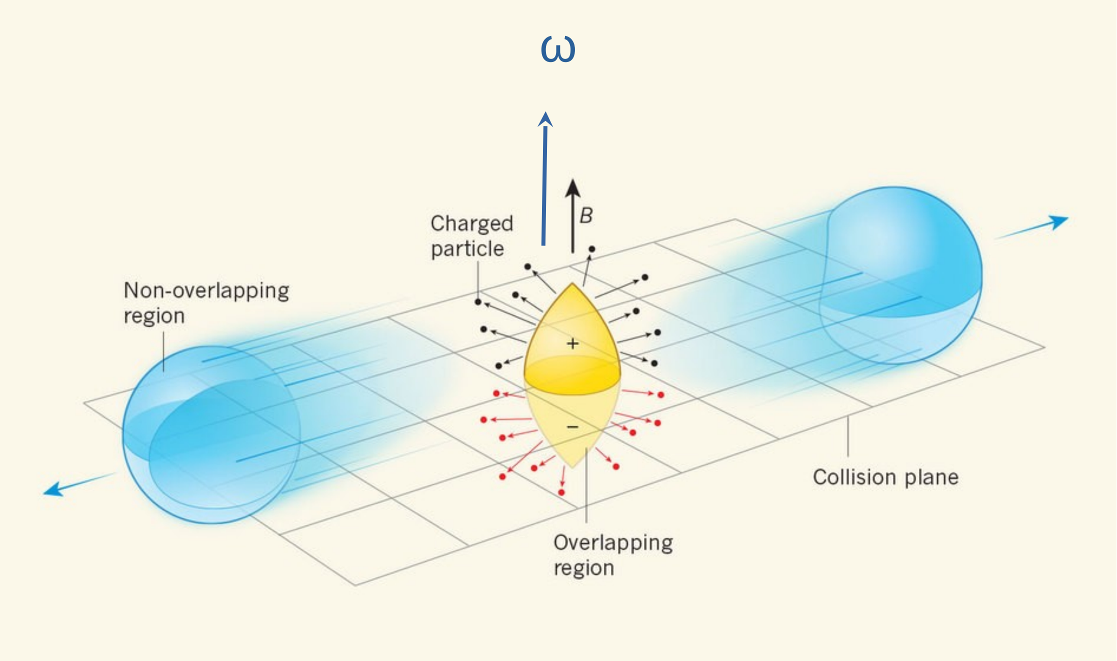


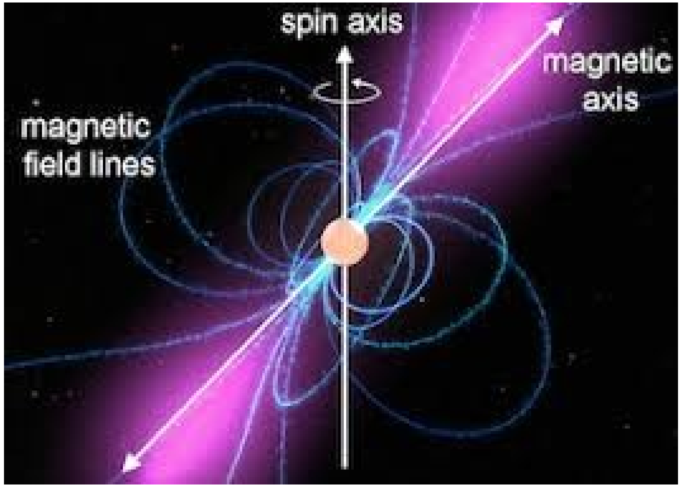
Spin polarization as momentum-unintegrated CVE



CME leads to negative magnetoresistance in ZrTe5

Interplay of magentic and vorticity fields



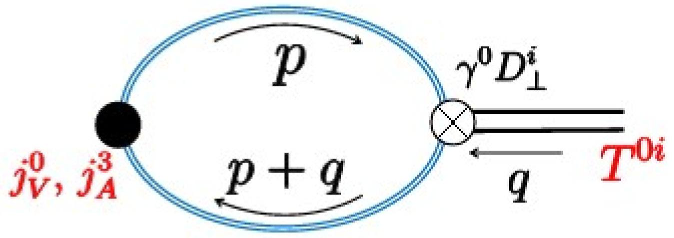


Magneto-vorticity effect

$$J^0 = \frac{C}{2} e(\mathbf{B}\cdot\boldsymbol{\omega})$$

$$\mathbf{J}_5 = \frac{C}{2}|e|(\mathbf{B}\cdot\boldsymbol{\omega})\hat{\mathbf{B}}$$

$$\mathbf{B} \parallel \boldsymbol{\omega}$$



Hattori, Yin, PRL 2017

Interpretation as CSE

$$\mathbf{J}_5 = C\mu e\mathbf{B} = C\frac{J^0}{\chi}e\mathbf{B} \qquad \chi = C|e|B$$

susceptibility from lowest Landau level state

Alternative interpretation

$$J^0 = -\boldsymbol{\nabla}\cdot\mathbf{P} - 2\mathbf{M}\cdot\boldsymbol{\omega}$$

Kovtun, JHEP 2016

$$J^0 = \frac{C}{2}e(\mathbf{B}\cdot\boldsymbol{\omega})$$

P: polarization

M: magnetization → sum of spin of LLL state

$$\mathbf{J}_5 = C\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)\boldsymbol{\omega}$$

$$\mathbf{J}_5 = \frac{C}{2}|e|(\mathbf{B}\cdot\boldsymbol{\omega})\hat{\mathbf{B}}$$

Interpretation as CVE?

Gravitational anomaly contribution?

MVE in strongly coupled magnetized plasma

Holographic model

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\Big\{R[g] + 12 - \frac{1}{4}(F^V)^2 - \frac{1}{4}(F^a)^2 + \epsilon^{MNPQR}A_M \times \Big[\frac{1}{3}\alpha(F^a)_{NP}(F^a)_{QR} + \alpha(F^V)_{NP}(F^V)_{QR} + \lambda R^Y{}_{XNP}R^X{}_{YQR}\Big]\Big\}$$

$\alpha,\ \lambda$ fixed using chiral/gravitational anomalies

Landsteiner et al JHEP (2011)

dictionary

$$V_\mu \leftrightarrow J^\mu$$

$$A_\mu \leftrightarrow J_5^\mu$$

$$g_{\mu\nu} \leftrightarrow T^{\mu\nu}$$

Neutral magnetic brane background

$$ds^2 = 2drdt - f(r)dt^2 + e^{2W_T(r)}(dx^2 + dy^2) + e^{2W_L(r)}dz^2$$

$$V = Bxdy \Rightarrow \vec{B} = B\hat{z}$$

D'Hoker, Kraus JHEP (2009)

$$f(r \simeq r_h) = 0 + f'(r_h)(r - r_h) + \cdots \qquad T = \left.\frac{\partial_r(f(r))}{4\pi}\right|_{r=r_h}$$

Dual to neutral magnetized plasma

Analytic background known for weak B
Numerical background available for arbitrary B

Metric induced vorticity

$$J^t = \xi(\mathbf{B}\cdot\boldsymbol{\omega}) \qquad\qquad \mathbf{J}_5 = \sigma\boldsymbol{\omega}$$

boundary metric $\quad ds^2_{\mathrm{M}} = -dt^2 + d\vec{x}^2 + 2h_{ti}(t,\vec{x})dtdx^i$

vorticity $\quad \omega^i = \frac{1}{2}\epsilon^{ijk}\nabla_j u_k = \frac{1}{2}\epsilon^{ijk}\partial_j h_{tk}$

$$\xi = \frac{2}{B}\lim_{q\to 0}\frac{\langle J^t T^{ty}\rangle}{iq}, \qquad\qquad \sigma = 2\lim_{q\to 0}\frac{\langle J_5^z T^{ty}\rangle}{iq}.$$

Limits taken in a static state

Uniqueness of static state

Static state fixed dynamically!

Turn on vorticity adiabatically, time-dependence necessary

bulk perturbation

$$\delta(ds^2) = 2e^{W_T(r)}\left[\delta g_{ty}(r,t,x)dtdy + \delta g_{xy}(r,t,x)dxdy\right],$$

$$\delta V = \delta V_t(r,t,x)dt + \delta V_x(r,t,x)dx, \qquad \delta A = \delta A_z(r,t,x)dz,$$

Adiabatic limit $\omega \to 0$.

Analytic results for weak B

$$J^t = \xi(\mathbf{B}\cdot\boldsymbol{\omega})$$

$$\xi = \frac{128}{3}(12\log 2 - 5)\alpha\lambda - 2\log r_h - 1 + \mathcal{O}(B/T^2)$$

anomalous   non-anomalous

$$\mathbf{J}_5 = \sigma\boldsymbol{\omega}$$

$$r_h = 4\pi T$$

$$\sigma = r_h^2\left[64\lambda - \frac{32\lambda B}{3r_h^2} + \mathcal{O}(B^2/T^4)\right]$$

support CVE interpretation

Non-anomalous contribution agrees with MHD

$$J^t = \xi(\mathbf{B}\cdot\boldsymbol{\omega})$$

$$\xi = \frac{128}{3}(12\log 2 - 5)\alpha\lambda - 2\log r_h - 1 + \mathcal{O}(B/T^2)$$

$$J^t = 2\left(M_{\omega,\mu}\mathbf{B}\cdot\boldsymbol{\omega} - 2p_{,B^2}\mathbf{B}\cdot\boldsymbol{\omega}\right)$$

Kovtun, Hernandez, JHEP 2017

energy shift $2\mathbf{M}\cdot\boldsymbol{\omega}$

susceptibilities

$$2p_{,B^2} \equiv 2\frac{\partial p}{\partial(B^2)} \qquad 2p_{,B^2} = \log r_h \to \log\frac{4\pi T}{M}$$

M renormalization scale

$$M_\omega = \frac{\partial p}{\partial(\mathbf{B}\cdot\boldsymbol{\omega})} \qquad M_\omega = -\frac{1}{2}\mu$$

Scheme dependence

$$\Delta S_{\text{c.t.}} = -\frac{1}{2\kappa^2}\int d^4x\sqrt{-\gamma}\left(\frac{a}{4}\left(F^V\right)_{\mu\nu}\left(F^V\right)^{\mu\nu}\right)$$

$$\Delta J^\mu = -\frac{a}{2\kappa^2}\sqrt{-\gamma}\nabla_\nu F^{\mu\nu}$$

$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}u^\rho B^\sigma + E_\mu u_\nu - E_\nu u_\mu$$

$$\longrightarrow \quad \Delta J^t \sim a(\mathbf{B}\cdot\boldsymbol{\omega})$$

shift magnetic susceptibility $2p_{,B^2} \equiv 2\frac{\partial p}{\partial(B^2)}$

MVE in weakly coupled magnetized plasma

classical kinetic theory

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}} + Q(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_{\mathbf{p}})\, f(X,\mathbf{p}) = C[f],$$

chiral kinetic theory

$$\big[(1+\hbar Q\boldsymbol{\Omega}\cdot\mathbf{B})\,\partial_t + (\mathbf{v} + \hbar Q\mathbf{E}\times\boldsymbol{\Omega} + \hbar Q(\mathbf{v}\cdot\boldsymbol{\Omega})\mathbf{B})\cdot\nabla_{\mathbf{x}}$$
$$+ (Q\mathbf{E} + Q\mathbf{v}\times\mathbf{B} + \hbar Q^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega})\cdot\nabla_{\mathbf{p}}\big]\, f(X,\mathbf{p}) = C[f].$$

Berry curvature $\boldsymbol{\Omega} = \pm\frac{\hat{\mathbf{p}}}{2\mathbf{p}^2}$

Son, Yamamoto, PRL(2012)
Stephanov, Yin, PRL (2012)
Gao et al, PRL (2012)
Hidaka, Pu, Yang, PRD (2017)
+many more

Wigner function formalism

$$S^{<}_{ab}(x_1,x_2) \;=\; \left\langle \chi^{\dagger}_{b}(x_2)\chi_a(x_1)\right\rangle$$ for right-handed fermion

$$\widetilde{S}(X,y) = U\left(X,x_1\right) S(x_1,x_2) U\left(x_2,X\right) \qquad U(x_1,x_2) = \mathcal{P}\exp\left[-i\frac{1}{\hbar}\int_{x_2}^{x_1} dz\cdot A(z)\right]$$

Wigner function

$$\widetilde{S}(X,p) \;=\; \int d^4y \exp\left(\frac{i}{\hbar}p\cdot y\right)\widetilde{S}(X,y) \qquad X = \frac{1}{2}(x_1+x_2),\; y = x_1 - x_2,$$

assume weak off-equilbrium $\hbar\partial_X \ll p$

Wigner function solved by $\hbar$ expansion

CKT with free particle vs Landau level basis

$$\widetilde{S} = \bar{\sigma}_\mu j^\mu$$

$$\rightarrow \begin{cases} \Pi_\mu j^\mu = 0, \\ \Delta_\mu j^\mu = 0, \\ \Pi^\mu j^\nu - \Pi^\nu j^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Delta_\rho j_\sigma, \end{cases}$$

$$\Delta_\mu = \partial_\mu - \frac{\partial}{\partial p_\nu} F_{\mu\nu};$$

$$F_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} u^\rho B^\sigma + E_\mu u_\nu - E_\nu u_\mu$$

free particle basis

$$\Pi_\mu = p_\mu$$

$$B^\mu \sim E^\mu \sim O(\partial)$$

Vasak, Gyulassy, Elze, (1987)

Landau level basis

$$\Pi_\mu = p_\mu - \frac{1}{12}\frac{\partial^2}{\partial p_\nu \partial p_\lambda}\frac{\partial}{\partial x^\lambda} F_{\mu\nu}$$

$B^\mu \sim O(\partial^0)$ $E^\mu \sim O(\partial)$

regime of MHD

SL, Yang, PRD (2020)

Lowest Landau level solution

$$j^{\mu}_{(0)} = (u+b)^{\mu}\delta(p\cdot(u+b))f(p\cdot u)e^{\frac{p_T^2}{B}}$$

$B^{\mu} = Bb^{\mu}$ constant

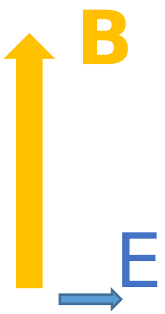
$$f(p\cdot u) = \frac{2}{(2\pi)^3}\sum_{r=\pm}\frac{r\theta(rp\cdot u)}{e^{r(p\cdot u-\mu_R)/T}+1}$$

Equilbrium distribution for LLL states

Sheng, Rishcke, Wang, Vasak, EPJA, 2018

Gao, SL, Mo, PRD 2020

I. Drift state



steady state with a drift velocity

$$u^{\mu}_{(1)} \equiv \frac{1}{2B}\epsilon^{\mu\nu\rho\sigma} f_{\nu\rho} b_{\sigma}$$

$$F_{\mu\nu} \to F_{\mu\nu} + f_{\mu\nu}$$

$$u^{\mu} \to u^{\mu}_{\mathcal{D}} \equiv (u + u_{(1)})^{\mu}$$

$$j^{\mu}_{(0)} + j^{\mu}_{(1)\mathcal{D}} = (u_{\mathcal{D}} + b)^{\mu} \delta(p \cdot (u_{\mathcal{D}} + b)) f(p \cdot u_{\mathcal{D}}) e^{(p^2 - (p \cdot u_{\mathcal{D}})^2 + (p \cdot b)^2)/B}$$

Matching drift state with MHD

$$J^\mu(X,p) = \int \frac{d^4p}{(2\pi)^4} j^\mu \qquad T^{\mu\nu}(X) = \int \frac{d^4p}{(2\pi)^4} p^{\{\mu} j^{\nu\}}(X,p)$$

matching with MHD structure

Kovtun, Hernandez, JHEP 2017
Grozdanov et al, PRD 2017
Hongo, Hattori, JHEP 2021

$$J^\mu = \mathcal{N} u^\mu + \mathcal{J}^\mu$$

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$\mathcal{J}^\mu_{(1)} = -\alpha_{BB,\mu}\epsilon^{\mu\nu\rho\sigma} u_\nu E_\rho B_\sigma,$$

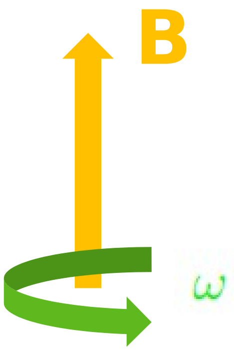
$$\mathcal{Q}^\mu_{(1)} = \left(M_{\omega,\mu} + 2p_{,B^2}\right)\epsilon^{\mu\nu\rho\sigma} u_\nu E_\rho B_\sigma$$

$\Rightarrow$

$$M_\omega^{\mathcal{D}} = -\frac{\mu}{8\pi^2} - \frac{\xi}{2\pi^2 B}$$

$$\xi \equiv \frac{1}{3}\mu\left(\mu^2 + \pi^2 T^2\right)$$

II. Vortical state



Ambiguity of static state also exists

$$j^{\mu}_{(0)} = (u+b)^{\mu}\delta(p\cdot(u+b))f(p\cdot u)e^{\frac{p_T^2}{B}}$$

$$f \to f + \delta f \qquad \delta f \sim O(\partial)$$

$$\begin{aligned} j^{\mu}_{(1)\mathcal{V}} = (u+b)^{\mu}\Bigg[-\frac{\omega}{3}\left(\frac{p_T^2}{B}+1\right)\delta'\left(p\cdot(u+b)\right) f\left(p\cdot u\right) + \frac{2\omega p_T^2}{3B}\delta\left(p\cdot(u+b)\right) f'\left(p\cdot u\right) \\ - \frac{2\omega p_T^2}{B^2} p\cdot u\,\delta\left(p\cdot(u+b)\right) f\left(p\cdot u\right)\Bigg] e^{\frac{p_T^2}{B}} + \frac{\omega p_T^{\mu}}{B}\delta\left(p\cdot(u+b)\right) f\left(p\cdot u\right) e^{\frac{p_T^2}{B}}. \end{aligned}$$

Matching vortical with MHD

$$J^\mu(X,p) = \int \frac{d^4p}{(2\pi)^4} j^\mu \qquad T^{\mu\nu}(X) = \int \frac{d^4p}{(2\pi)^4} p^{\{\mu} j^{\nu\}}(X,p)$$

matching with MHD structure

$$J^\mu = \mathcal{N} u^\mu + \mathcal{J}^\mu$$

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

$$\mathcal{N}_{(1)} = f_{\mathcal{N}} = -2\left(2p_{,B^2} + M_{\omega,\mu}\right) B\omega,$$

$$\mathcal{E}_{(1)} = f_{\mathcal{E}} = -2\left(TM_{\omega,T} + \mu M_{\omega,\mu} - 2M_\omega\right) B\omega,$$

$$\mathcal{P}_{(1)} = f_{\mathcal{P}} = \frac{2}{3}\left(M_\omega + 4M_{\omega,B^2} B^2\right) B\omega,$$

$$\mathcal{T}_{(1)} = f_{\mathcal{T}} = -\frac{4}{3}\left(M_{\omega,B^2} B^2 + M_\omega\right) B\omega,$$

→

$$M_\omega^{\mathcal{V}} = \frac{\mu}{8\pi^2} + \frac{\#\xi}{B}$$

Vacuum state ambiguity

Magneto-vortical susceptibility $M_\omega = \frac{\partial p}{\partial(\mathbf{B}\cdot\boldsymbol{\omega})}$

assumes state unchanged as vorticity is turned on

$$M_\omega^{\mathcal{D}} = -\frac{\mu}{8\pi^2} - \frac{\xi}{2\pi^2 B}$$

$$M_\omega^{\mathcal{V}} = \frac{\mu}{8\pi^2} - \frac{\xi}{2\pi^2 B}$$

energy density raised by $\frac{\mu B\omega}{2\pi^2}$ due to vorticity

$$\frac{\mu B\omega}{2\pi^2} = \omega \times \frac{\mu B}{2\pi^2} = \Delta\mu \times n_{\mathrm{LLL}}$$

$$j^\mu_{\mathrm{vac}} = \omega(u \pm b)^\mu \delta\left(p \cdot (u \pm b)\right) f'\left(p \cdot u\right) e^{\frac{p_T^2}{B}}$$

MVE from kinetic theory

$$\Delta J^0 = -2(2p_{,B^2} + M^{\mathcal{D}}_{\omega,\mu})B\omega = \left(\frac{B}{4\pi^2} + \frac{\chi}{\pi^2}\right)\boldsymbol{\omega}\cdot\mathbf{b}$$

MHD with unshifted vacuum

$$\Delta J^0 = -2(2p_{,B^2} + M^{\mathcal{V}}_{\omega,\mu})B\omega - J^0_{\text{vac}} = \left(\frac{B}{4\pi^2} + \frac{\chi}{\pi^2}\right)\boldsymbol{\omega}\cdot\mathbf{b}$$

MHD with shifted vacuum

$$\Delta \mathbf{J}_A = \left(\frac{B}{4\pi^2} + \frac{\chi}{\pi^2}\right)\boldsymbol{\omega}$$

$$\chi \equiv \frac{\mu^2}{2} + \frac{\pi^2 T^2}{6}$$

Summary & Outlook

- Magneto-vortical effect as a new anomalous effect
- Scheme dependence of MVE seen in interacting theory
- Vacuum ambiguity fixed dynamically in strongly coupled plasma
- Vacuum ambiguity fixed by matching with MHD in weakly coupled plasma
- Generalization to higher LL? Collisional effect?

Thank you!