Wilson Loop Duality and OPE for Form Factors of 1/2-BPS Operators

2023 Theoretical Physics Seminar
Shing-Tung Yau Center of Southeast University

Benjamin Basso - LPENS

Based on recent work with Alexander Tumanov

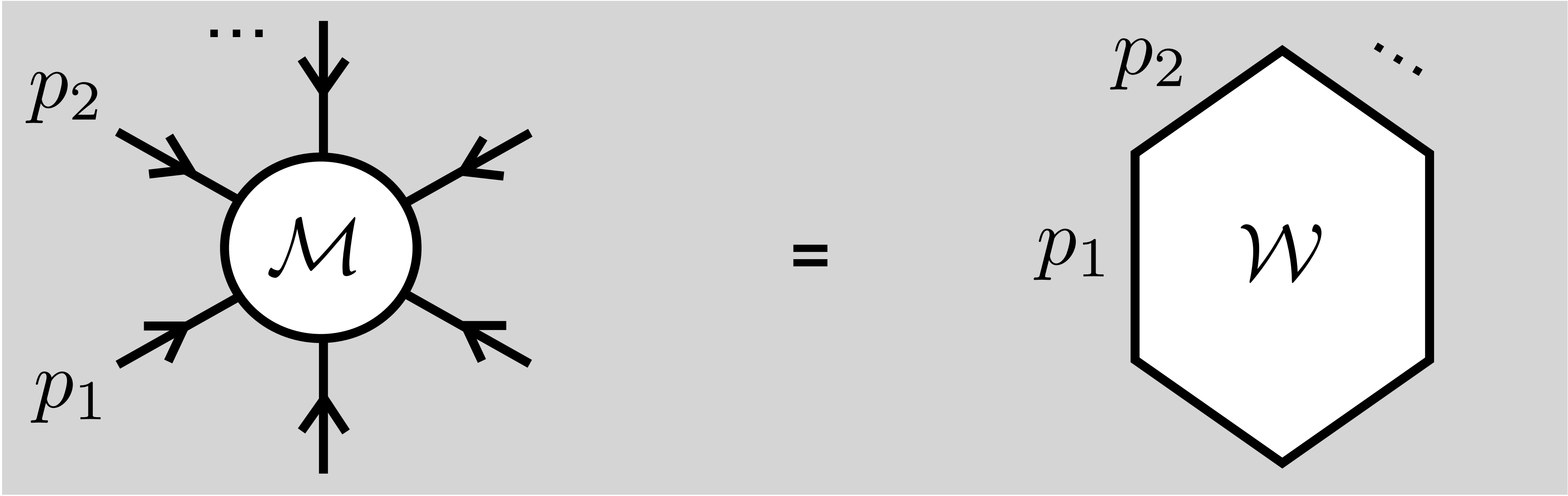
Solving N=4 Super-Yang-Mills

- Maximally supersymmetric Yang-Mills theory in 4 dimensions
- Lots of symmetries: conformal invariance, supersymmetry, S-duality, …
- AdS/CFT correspondence: Duality with superstring theory on AdS5*S5
- Integrability: The theory is believed to be "exactly solvable" in the large Nc (planar) limit
- Wonderful laboratory for studying gauge/string duality and uncovering hidden structures in gauge theories
- Lots of progress with correlation functions: exact spectrum of scaling dimensions and methods at higher points
- Important developments in calculation of on-shell quantities such as scattering amplitudes

Scattering amplitudes / Wilson loops duality

- **Duality** between amplitudes and null polygonal Wilson loops

$$\text{“} \log \mathcal{A}_n = \log W_n \text{”}$$



- Null momenta of amplitude map to null edges of WL

- Bonus: two copies of conformal groups (original and dual) - combine into a Yangian (manifestation of integrability)

- Dual conformal symmetry put severe constraints on amplitudes

[Alday,Maldacena]
[Drummond,Korchemsky,Sokatchev]
[Brandhuber,Heslop,Travaglini]
[Drummond,Henn,Korchemsky,Sokatchev]
[Drummond,Henn,Plefka]

  - 4- and 5-gluon amplitudes entirely fixed by dual conformal symmetry

  - Finite part of n > 5 amplitudes not fixed by functions of 3n-15 cross ratios (ratios of products of Mandelstam invariants / distances between cusps)

Progress report

- Duality is at the heart of many recent progress in scattering amplitudes

  Amplituhedron/Grassmannian, Amplitude Function Bootstrap, TBA, Pentagon OPE, …

- **Function Bootstrap**: Aim at constructing higher multiplicity amplitudes at higher loops in generic kinematics
- Combine physical/structural requirements on solutions with known boundary data (collinear/Regge limit, …)
  - 6-point amplitudes through 7 loops
  - 7-point amplitudes through 4 loops
  - 8-point amplitudes through 3 loops
  - n-point amplitudes through 2 loops (symbol)

[Bern,Dixon,Smirnov]
[Goncharov,Spradlin,Vergu,Volovich][Dixon,Drummond,Henn]
[Caron-Huot,Dixon,Drummond,Duhr,vonHippel,McLeod,Pennington]
[Caron-Huot,Dixon,Dulat,vonHippel,McLeod,Papathanasiou]
[Golden,Spradlin]
[Drummond,Papathanasiou,Spradlin]
[Dixon,Drummond,McLeod,Harrington,Papathanasiou,Spradlin]
[Dixon,Liu]
[Golden,McLeod]
[Caron-Huot,He]
[Li,Zhang]
[Caron-Huot]

- **Integrable Bootstrap**: Aim at constructing amplitudes at finite coupling as a systematic (OPE) expansion

[Alday,Gaiotto,Maldacena,Sever,Vieira]
[BB,Sever,Vieira]
[Belitsky]

What about other on-shell observables?

- Important class: Form factors of local operators - bridge between amplitudes and operators

$$F_{\mathcal{O}}(p_1,\ldots,p_n;q) = \int d^4x\, e^{-iqx}\langle p_1,\ldots,p_n|\mathcal{O}(x)|0\rangle$$

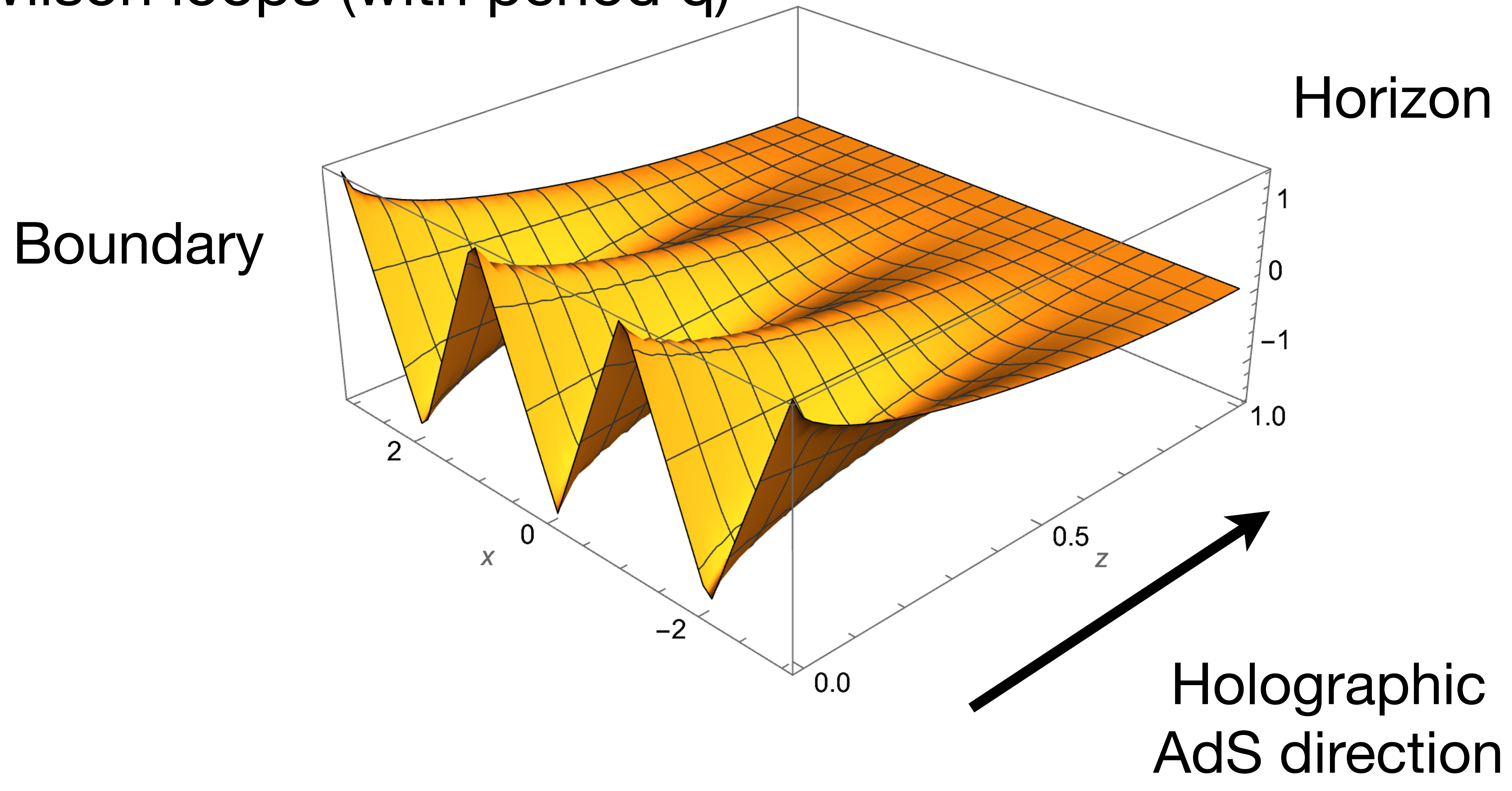
[Alday,Maldacena]
[Maldacena,Zhiboedov]
[Brandhuber,Spence,Travaglini,Yang]
[Brandhuber,Gurdogan,Mooney,Travaglini,Yang]
[Bork][Sever,Tumanov,Wilhelm]

- String Theory: T-duality, mapping UV into IR and vice versa
- It follows that WL cannot be closed when total momentum non zero $q = \sum_{i=1}^{n} p_i \neq 0$
- Hint at duality between form factors and null periodic Wilson loops (with period q)
- Lots of evidence gathered for operators in stress tensor super-multiplet

[Brandhuber,Spence,Travaglini,Yang]
[Brandhuber,Gurdogan,Mooney,Travaglini,Yang]
[Penante,Spence,Travaglini,Wen]
[Bork][Brandhuber,Penante,Travaglini,Wen]

- New integrable bootstrap (FFOPE) and function bootstrap were proposed to determine its form factors at finite / weak coupling

[Sever,Tumanov,Wilhelm]
[Dixon,Gurdogan.McLeod,Wilhelm]
[Dixon,Gurdogan.Liu,McLeod,Wilhelm]
[Guo,Wang,Yang]



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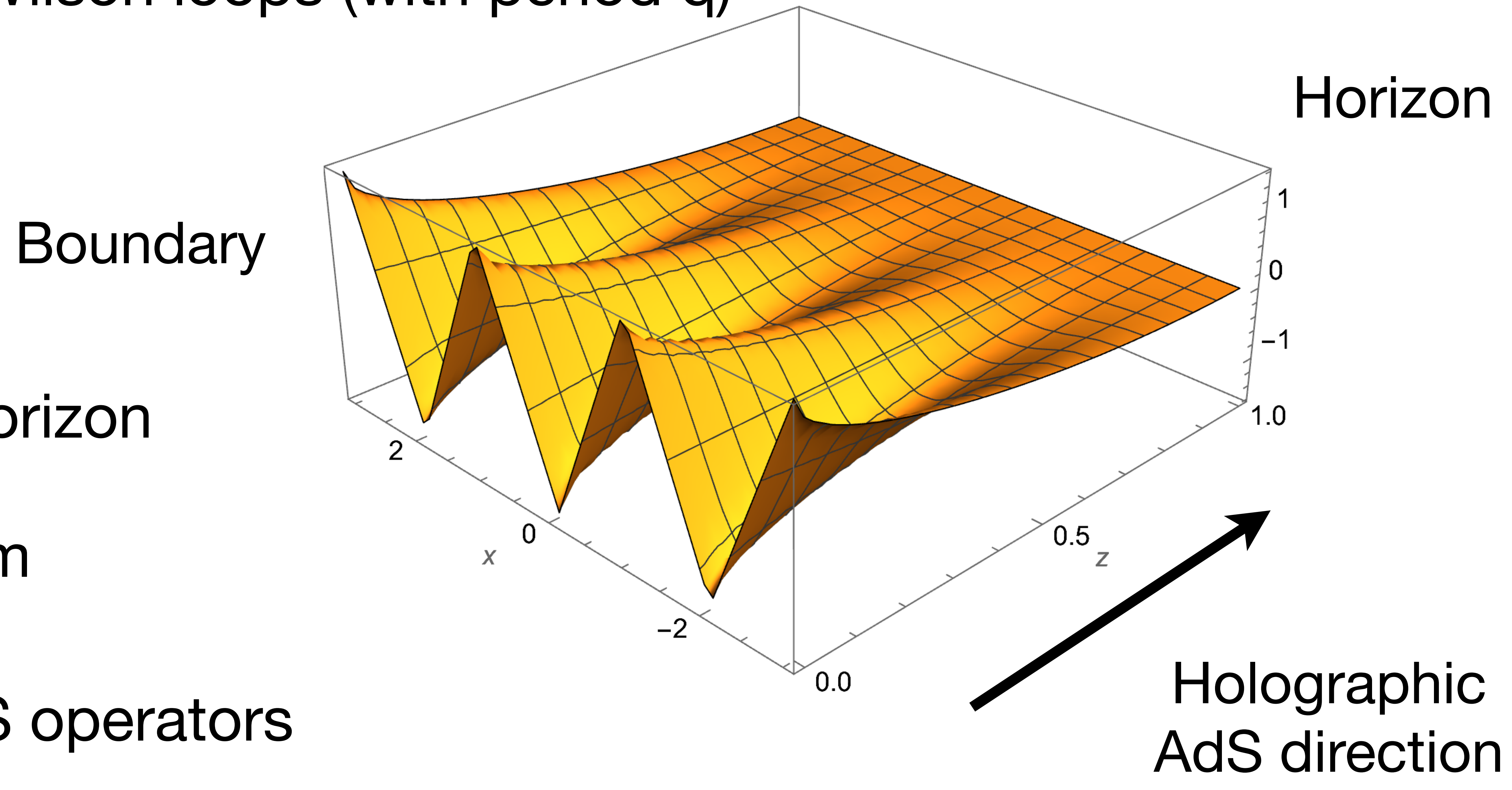
- Hint at duality between form factors and null periodic Wilson loops (with period q)
- More general operators?
- Expect
  - Local operators to map to states (BC) at the horizon
  - Integrable bootstrap should apply to all of them
- Natural step: develop WL duality and OPE for half-BPS operators



Plan

- Super form factors of half BPS operators
- Super Wilson loop duals
- FFOPE and integrable bootstrap
- Conclusion

Super Form Factors and Super Wilson Loops

States and operators

- On-shell state: gluons and super-partners (gauginos and scalars)

[Brandhuber,Gurdogan,Mooney,Travaglini,Yang]
[Penante,Spence,Travaglini,Wen]

- They may all be encoded in a single CPT invariant on-shell N = 4 superstate

[Nair]

$$\Phi(\lambda, \tilde{\lambda}, \tilde{\eta}) = G^{+} + \Psi_A \tilde{\eta}^A + \Phi_{AB}\tilde{\eta}^A\tilde{\eta}^B + \ldots + \tilde{\eta}^1 \ldots \tilde{\eta}^4 G^{-}$$

- $\lambda^{\alpha}\,,\tilde{\lambda}^{\dot{\alpha}}$ are spinor helicity variables and $\tilde{\eta}^{A=1,2,3,4}$ are Grassmann variables

- Eigenstate with momentum $p^{\alpha\dot{\alpha}} = \sigma_{\mu}^{\alpha\dot{\alpha}} p_{\mu} = \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}$ and super momentum $q^{\alpha A} = \lambda^{\alpha}\tilde{\eta}^{A}$

- 1/2-BPS operator: Chiral Primary Operator and its Q descendants $\mathcal{T}_k(\theta) = e^{\theta_{\alpha A} Q^{\alpha A}} \cdot \mathrm{Tr}\,\phi(0)^k$

- Stress-tensor supermultiplet (k=2) : $\mathcal{T}_2(\theta) = \mathrm{Tr}\,\phi(0)^2 + \ldots + (\theta)^4\,\mathrm{Tr}\,\mathcal{L}$

- CPO have R-charge k and scaling dimension k

Super form factors

[Brandhuber,Gurdogan,Mooney,Travaglini,Yang]
[Penante,Spence,Travaglini,Wen]

- Super form factors are defined as the super-Fourier transform of matrix elements of $\mathcal{T}_k(x)$

$$\mathcal{F}_{k,n}(1,\ldots,n;q,\gamma) = \int d^4x d^4\theta e^{-iqx-\gamma\theta} \langle \Phi_1 \ldots \Phi_n | \mathcal{T}_k(x) | 0 \rangle$$

- Unlike scattering amplitudes, for form factors the (super) momentum is generically non zero

$$q^{\alpha\dot{\alpha}} = \sum_{i=1}^{n} \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \neq 0 \qquad \gamma^{\alpha +} = \sum_{i=1}^{n} \lambda_i^\alpha \tilde{\eta}_i^{a'} \neq 0 \qquad \gamma^{\alpha -} = \sum_{i=1}^{n} \lambda_i^\alpha \tilde{\eta}_i^{a} = 0$$

- Split R-indices with a = 1,2 for supercharges annihilating the operators and a' = 3,4 for the remaining ones
- SUSY Ward identities put constraints on the super form factors which must take the form

$$\mathcal{F}_{k,n} = \frac{\delta^{(4)}(q - \sum_i \lambda_i \tilde{\lambda}_i)\delta^{(2|2)}(\gamma^+ - \sum_i \lambda_i \tilde{\eta}_i^+)\delta^{(2|2)}(\sum_i \lambda_i \tilde{\eta}_i^-)}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle} \times W_{k,n} \quad \text{where} \quad \langle ij \rangle = \epsilon_{\alpha\beta} \lambda_i^\alpha \lambda_i^\beta$$

- MHV form factors correspond to terms with lowest fermionic degree $W_{k,n}^{\text{MHV}} = \text{Poly}^{2(k-2)}(\tilde{\eta}_i^{a=1,2})$

Tree level data

- Explicit expressions for MHV tree form factors have been obtained for all half-BPS operators $\mathrm{Tr}\,\Phi(0)^k$

- Simplest examples:

[Brandhuber,Gurdogan,Mooney,Travaglini,Yang]
[Penante,Spence,Travaglini,Wen]

$$W^{\rm tree}_{k=2,n} = 1$$

$$W^{\rm tree}_{k=3,n} = \sum_{1\leqslant i<j\leqslant n} \langle ij\rangle\, \tilde{\eta}_i^- \cdot \tilde{\eta}_j^-$$

(cyclic when $\gamma^- = 0$)

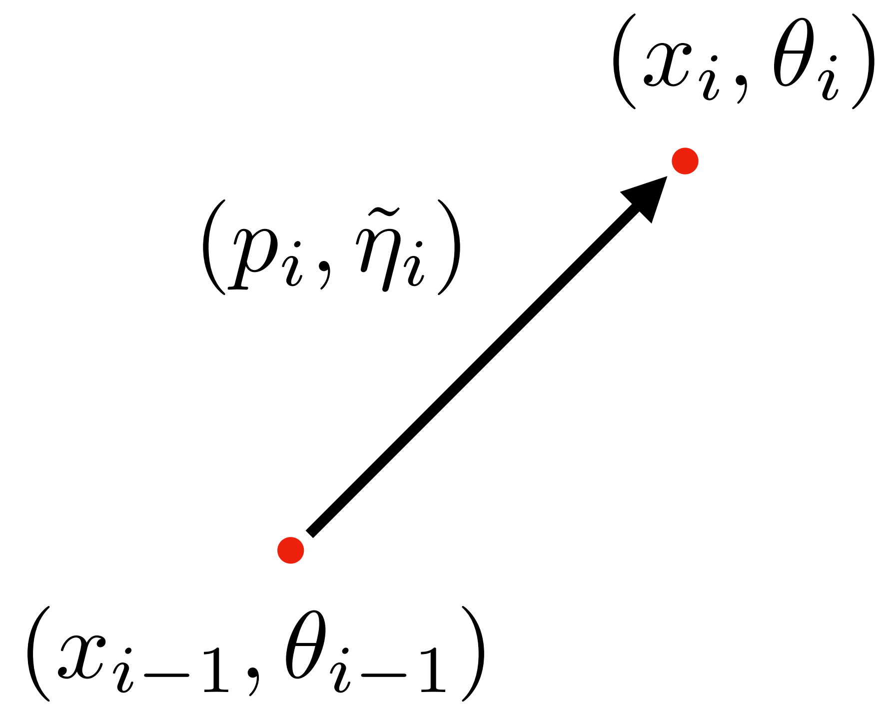
- Higher-k form factors are given by higher polynomials in the Lorentz $\times$ R-invariant products

$$\langle ij\rangle = \epsilon_{\alpha\beta}\lambda_i^\alpha\lambda_j^\beta \qquad\qquad \tilde{\eta}_i^- \cdot \tilde{\eta}_j^- = \frac{1}{2}\epsilon_{ab}\tilde{\eta}_i^a\tilde{\eta}_j^b$$

- General formula at higher k is most easily obtained by using BCFW-like recursion relations (given later on)

Mapping to dual coordinates

- Gain insight into dual Wilson loop by performing transformation to dual coordinates



- (Super) momenta map to (super) coordinates of the cusps of a null polygon

$$\lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} = x_i^{\alpha\dot{\alpha}} - x_{i-1}^{\alpha\dot{\alpha}} \qquad \lambda_i^\alpha \tilde{\eta}^A = \theta_i^{\alpha A} - \theta_{i-1}^{\alpha A}$$

- Alternatively one may specify the null polygon using (super) momentum twistors

[Drummond,Henn,Korchemsky,Sokatchev]
[Hodges]

$$\mathcal{Z}_i = \begin{pmatrix} \lambda_i^\alpha \\ \mu_i^{\dot{\alpha}} \\ \eta_i^A \end{pmatrix} = \begin{pmatrix} \lambda_i^\alpha \\ \lambda_{i\alpha} x_i^{\alpha\dot{\alpha}} \\ \lambda_{i\alpha}\theta_i^{\alpha A} \end{pmatrix} \in \mathbb{P}^{3|4}$$

- With non-local transformations

$$\tilde{\lambda}_i^{\dot{\alpha}} = \frac{\langle i i+1\rangle \mu_{i-1}^{\dot{\alpha}} + \langle i+1 i-1\rangle \mu_i^{\dot{\alpha}} + \langle i-1 i\rangle \mu_{i+1}^{\dot{\alpha}}}{\langle i-1 i\rangle\langle i i+1\rangle} \qquad \tilde{\eta}_i^A = \frac{\langle i i+1\rangle \eta_{i-1}^A + \langle i+1 i-1\rangle \eta_i^A + \langle i-1 i\rangle \eta_{i+1}^A}{\langle i-1 i\rangle\langle i i+1\rangle}$$

Dual superconformal symmetry

- Similarly to amplitudes, MHV form factors exhibit dual super-conformal symmetry
- Precisely, they are invariant under an SL(2|2) subgroup of transformations acting on the dual variables

$$z_i = \begin{pmatrix} \lambda_i^\alpha \\ \eta_i^a \end{pmatrix} \quad \text{with} \quad \begin{matrix} \alpha = 1,2 \\ a = 1,2 \end{matrix}$$

- MHV form factors may be expressed in terms of SL(2|2) R-invariants

$$(ijk) \equiv \frac{\prod_{a=1}^{2}(\langle ij\rangle \eta_k^a + \text{cyclic})}{\langle ij\rangle\langle jk\rangle\langle ki\rangle}$$

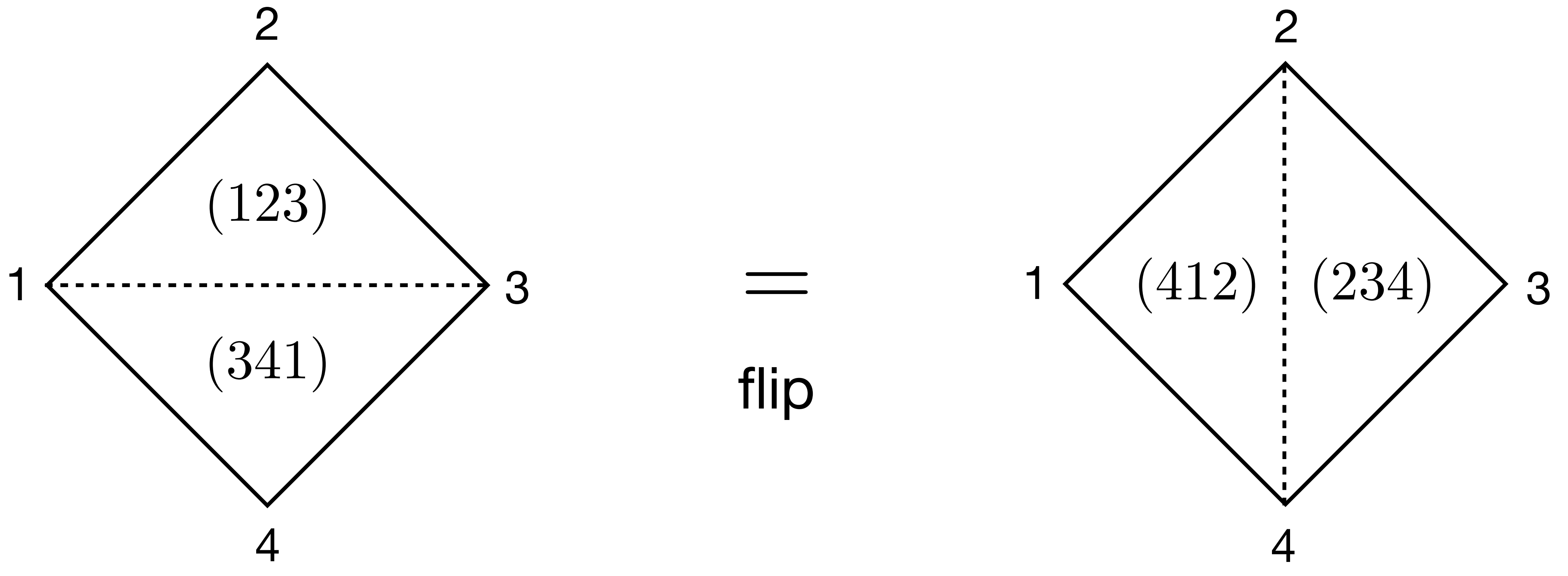
- Tree form factors for k = 3 may be written concisely for any any number of points

$$W_{k=3,n}^{\text{tree}} = \sum_{i=2}^{n-1}(1ii+1)$$

m = 2 Amplituhedron

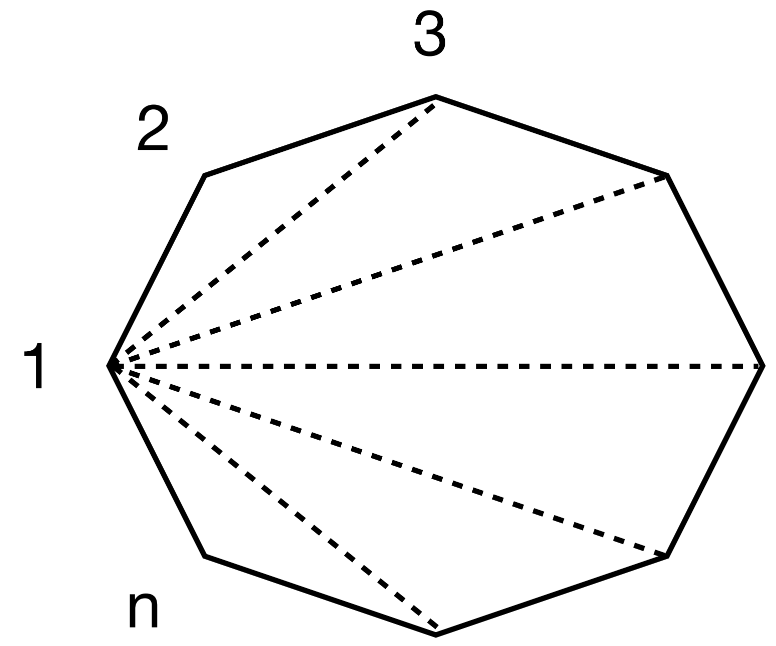
- R-invariants are subject to 4-term identity $(123) + (341) = (412) + (234)$

- It may be understood geometrically as an equivalence relation for two triangulations of a square



- One may see n-pt form factors as area of a convex n-gon where each tile is associated with an R-invariant

$$W^{\text{tree}}_{k=3,n} = \sum_{i=2}^{n-1} (1ii+1) \quad \Longleftrightarrow$$



Same structure observed recently for correlation functions [Caron-Huot,Coronado,Muhlmann]

- Form factors provide a realisation of the m=2 Amplituhedron (geometric reformulation of scattering amplitudes)

Higher-charge form factors

- Higher-charge form factors were conjectured to satisfy BCFW-like recursion relations [Penante,Spence,Travaglini,Wen]

- They relate form factors with different k and n

$$W_{k,n}^{\text{tree}}(1,\ldots,n) = W_{k,n-1}^{\text{tree}}(1,\ldots,n-1) + (n-1n1)W_{k-1,n-1}^{\text{tree}}(1,\ldots,n-1)$$

- This relation has a simple geometrical interpretation for k = 3

- General solution for any k can be written concisely

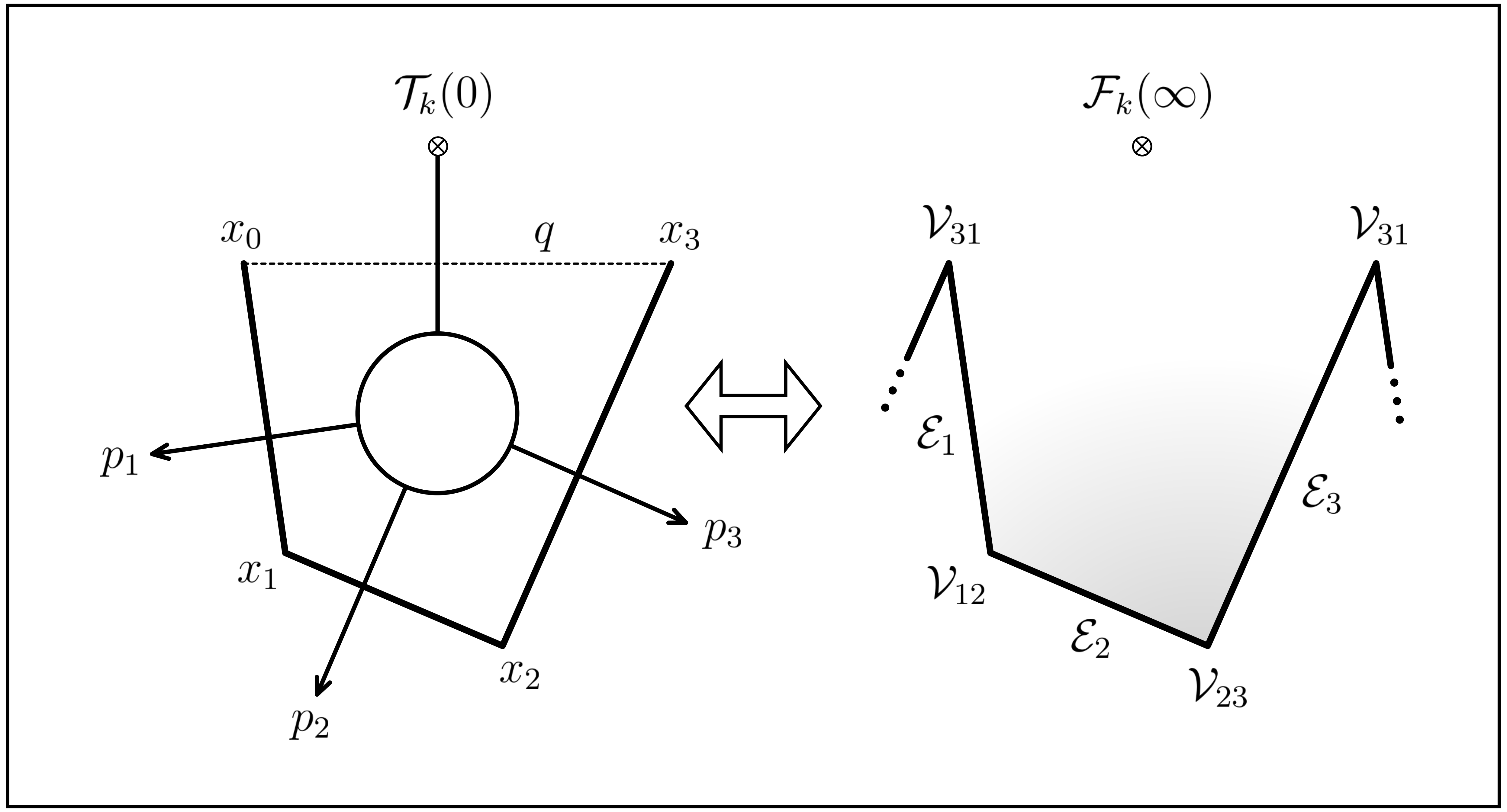
$$W_{k,n}^{\text{tree}} = \frac{1}{(k-2)!}\left[W_{3,n}^{\text{tree}}\right]^{k-2}$$

- Higher-charge form factors are higher-degree polynomials in R-invariants

- Formula is also known in the context of the m=2 Amplituhedron where k-2 plays the role of helicity degree

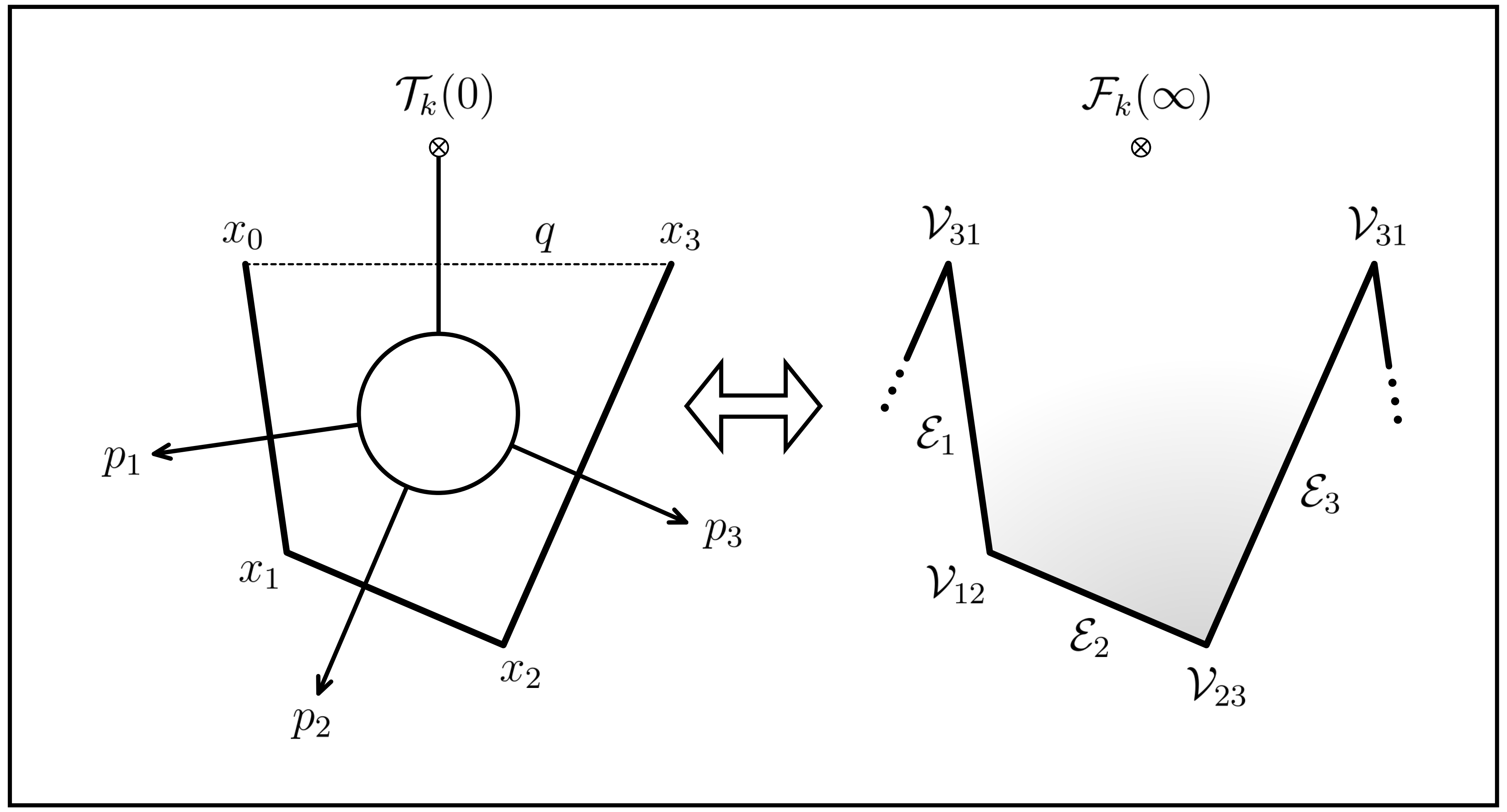
Super Wilson loop dual

- Can one recover this data from actual Wilson loop calculation? Precise Wilson loop dual?
- Total momentum is non zero: we should consider a periodic Wilson loop



- Futhermore, must be a super Wilson loop since MHV form factors depend on Grassmann variables

Dual operator



[Caron-Huot]
[Mason,Skinner]

- Null polygonal super Wilson loop

$$W_n = \frac{1}{N_c}\mathrm{Tr}\left[\cdots \mathcal{V}_{n1} e^{i\int_0^1 dt_1 \mathcal{E}_1(t_1)} \mathcal{V}_{12} e^{i\int_0^1 dt_2 \mathcal{E}_2(t_2)} \cdots \mathcal{V}_{n1}\cdots\right]$$

- Super-connection integrated along light-like edges

$$\mathcal{E}_i = p_i^{\alpha\dot{\alpha}} A_{\dot{\alpha}\alpha} + i\tilde{\lambda}_i^{\dot{\alpha}}\bar{\psi}_{\dot{\alpha}}\eta_i^a + \frac{i}{2\langle i i+1\rangle}\tilde{\lambda}_i^{\dot{\alpha}}\lambda_{i+1}^{\alpha} D_{\dot{\alpha}\alpha}\phi_{ab}\eta_i^a\eta_i^b$$

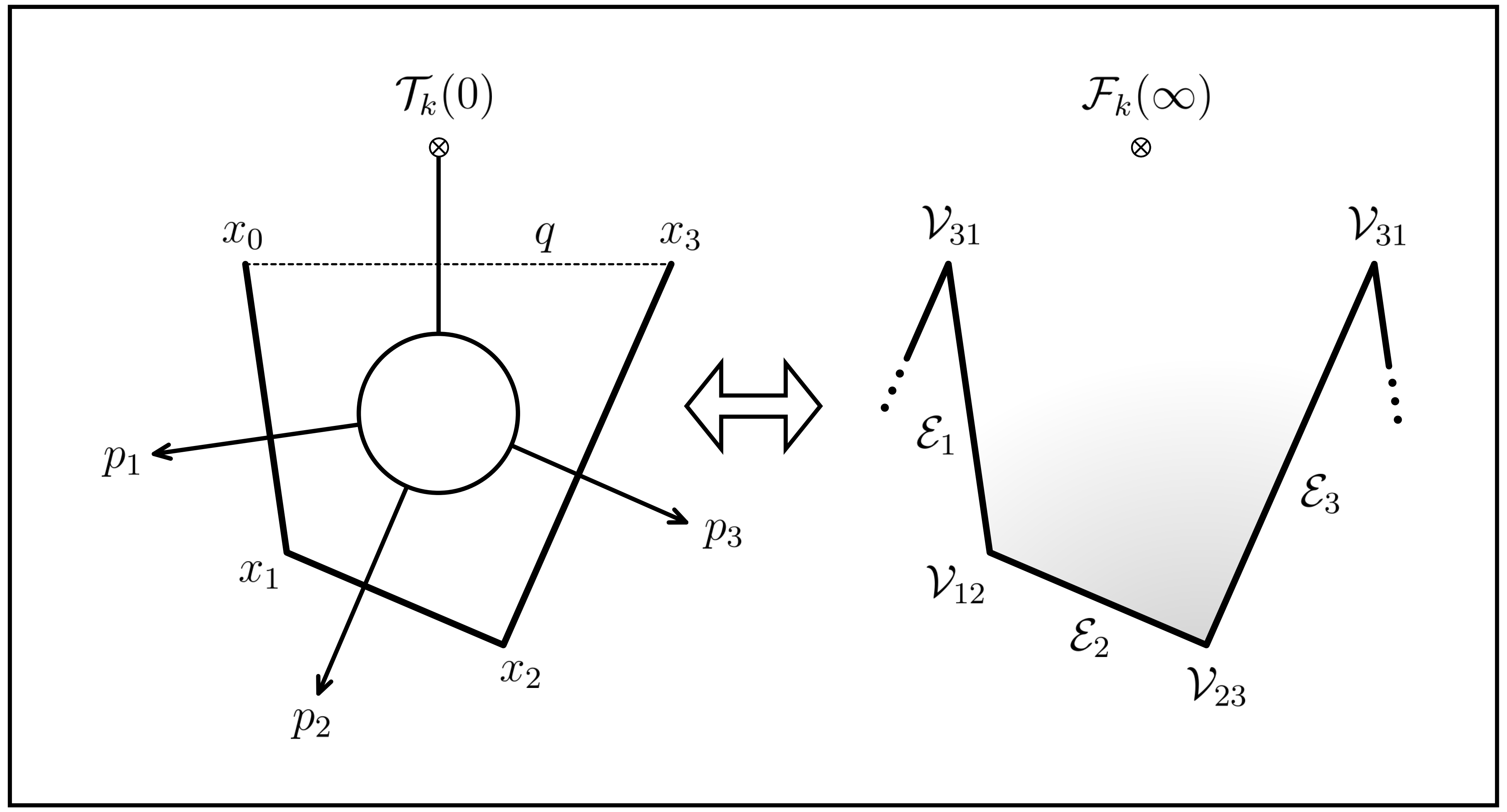
- Supplemented by vertex insertion at the cusp

$$\mathcal{V}_{ii+1} = 1 + \phi_{ab}\left[\frac{\eta_i^a\eta_{i+1}^b}{\langle i i+1\rangle} + \frac{\langle i-1 i+1\rangle \eta_i^a\eta_i^b}{\langle i-1 i\rangle\langle i i+1\rangle}\right] + \frac{1}{2}\,(\text{same thing})^2$$

- Defined such as to be annihilated by super-translations $\left[\mathcal{Q}_{\alpha a} + \sum_{i=1}^{n}\lambda_\alpha^i \frac{\partial}{\partial\eta_i^a}\right] W_n = 0$ (up to gauge transformation)

- For MHV form factors of half-BPS operators we may restrict ourselves to the SU(2) subsector with a, b = 1, 2

Dual state



- The super Wilson loop takes care of the external state in FF

$$W_n \quad \Leftrightarrow \quad \prod_{i=1}^{n} \Phi(\lambda_i, \tilde{\lambda}_i, \tilde{\eta}_i)$$

- To represent the local operator we must pick a dual state

$$\langle 1, \ldots, n | \operatorname{Tr} \phi(0)^k + \ldots |0\rangle \quad \Leftrightarrow \quad \langle \mathcal{F}_k | W_n | 0 \rangle$$

- Requirements: dual state must carry the right R-charge and be annihilated by (super) translations in dual space

$$\langle \mathcal{F}_k | \mathcal{P}_{\dot{\alpha}\alpha} = 0 \qquad \langle \mathcal{F}_k | \mathcal{Q}_{\alpha a} = 0$$

- This is because these symmetries are redundancies of the dual description

- Simplest choice: State composed of (k-2) **zero-momentum** scalar particles

$$\langle \mathcal{F}_k | = \langle \phi^{12}(0) \ldots \phi^{12}(0) |$$

- In particular, the dual state for k = 2 (stress tensor multiplet) is the vacuum state

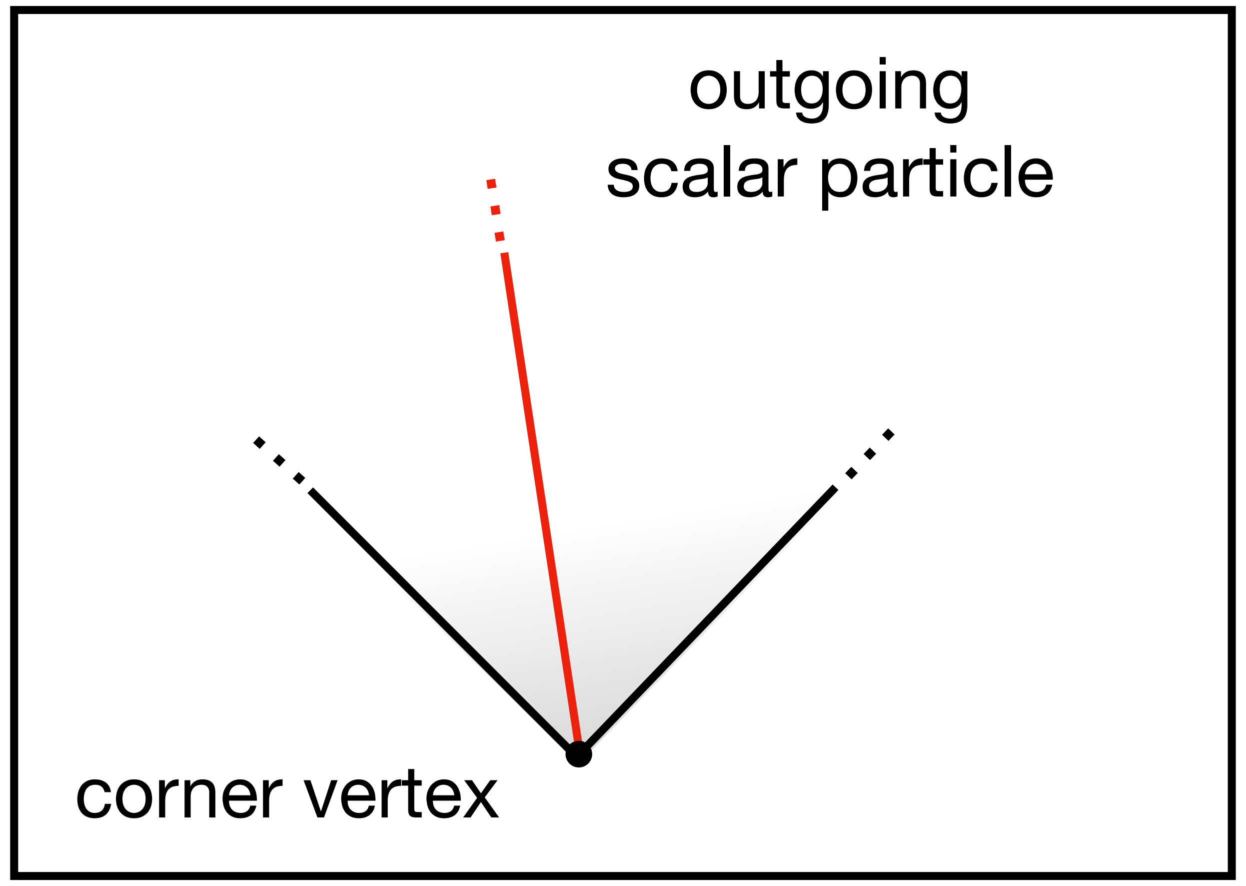
Tree level check

- We may perform a simple check of this conjecture at weak coupling: $\mathcal{E}_i = \mathcal{O}(g^2)$

$$W^{\text{tree}}_{n,k} = \langle \phi^{12}(0)\dots\phi^{12}(0)| \prod_{i=1}^{n-1} \mathcal{V}_{ii+1}|0\rangle$$

- Basic tree matrix element

$$\langle \phi^{12}(0)|\phi_{ab}(x)|0\rangle = \epsilon_{ab} \qquad \Leftrightarrow$$



- It immediately leads to

$$W^{\text{tree}}_{k=3,n} = \sum_{i=1}^{n-1} \langle \phi^{12}(0)|\mathcal{V}^{(\eta^2)}_{ii+1}|0\rangle = \sum_{i=1}^{n-1}\left[\frac{\eta_i^a\eta_{i+1}^b}{\langle ii+1\rangle} + \frac{\langle i-1i+1\rangle\eta_i^a\eta_i^b}{\langle i-1i\rangle\langle ii+1\rangle}\right]$$

- It agrees perfectly with the tree form factor data

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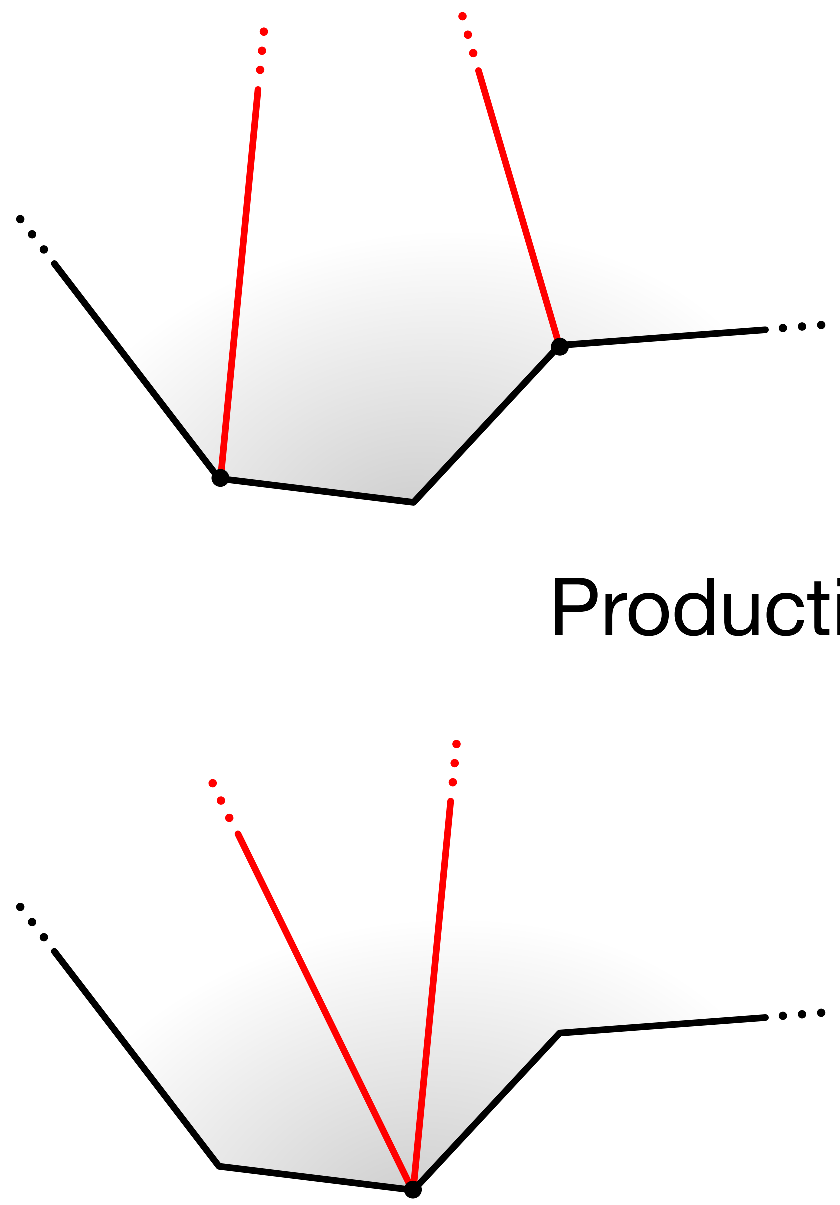
$$W_{n,k}^{\rm tree} = \langle \phi^{12}(0) \ldots \phi^{12}(0) | \prod_{i=1}^{n-1} \mathcal{V}_{ii+1} | 0 \rangle$$

- Basic tree matrix element

$$\langle \phi^{12}(0) | \phi_{ab}(x) | 0 \rangle = \epsilon_{ab}$$

- Similarly at higher k. E.g.

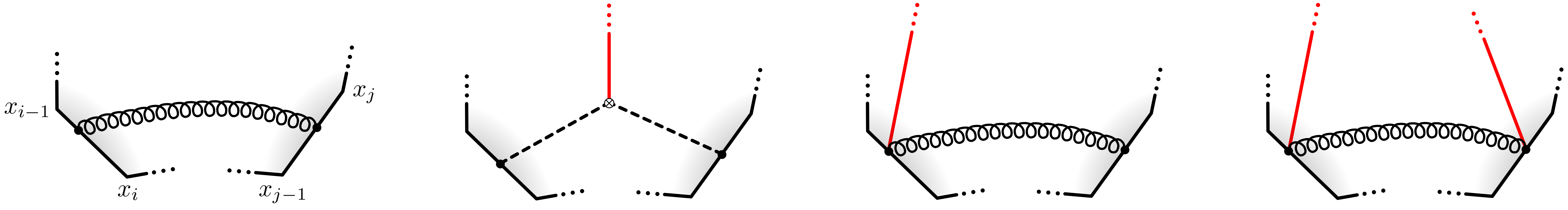
$$\Rightarrow \qquad W_{4,n}^{\rm tree} = \frac{1}{2} \left[W_{3,n}^{\rm tree} \right]^2$$



Production of 2 scalar state

Loop diagrams

- We may also perform a check at one loop
- Two main class of diagrams
- 1st class: exchange diagrams



- They generate long-range corrections to the corner vertex $\mathcal{V}_{ii+1} \to \mathcal{V}_{ij}$

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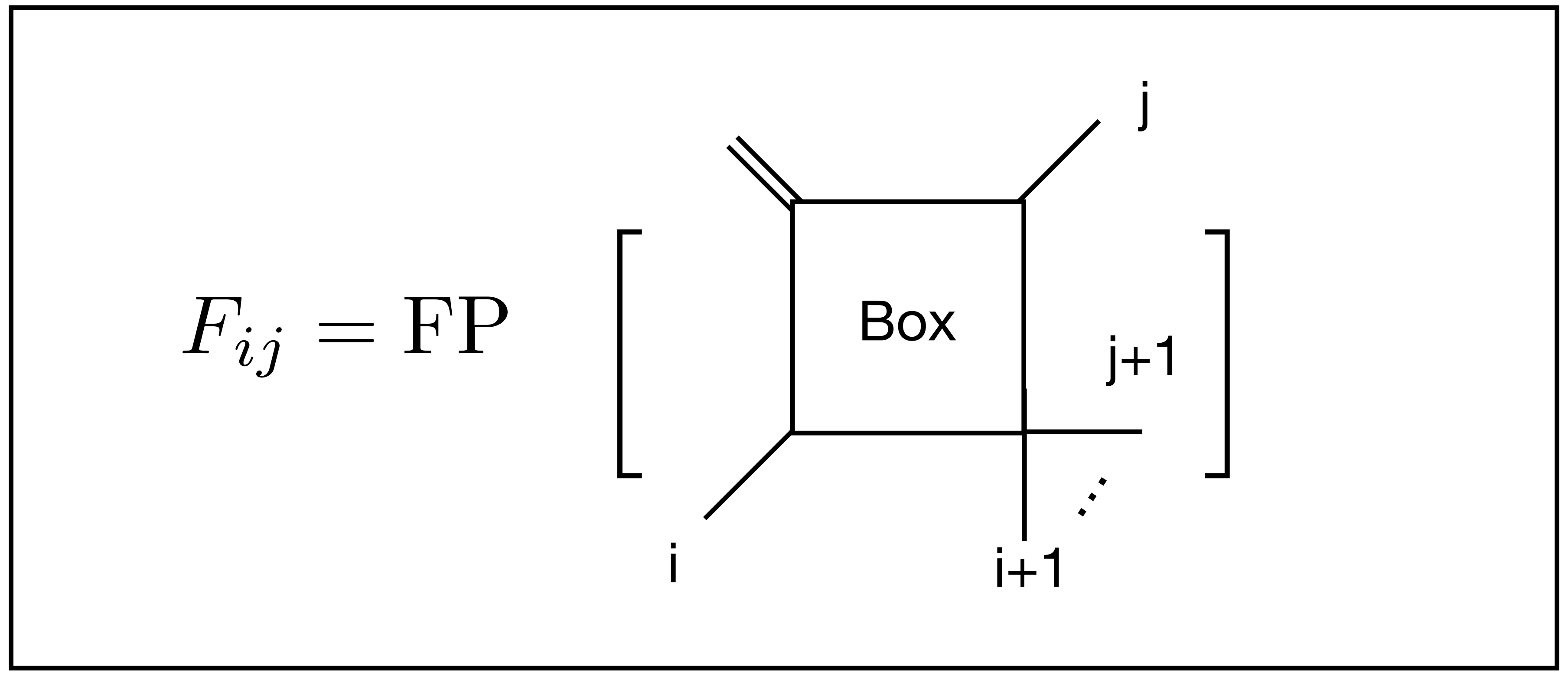
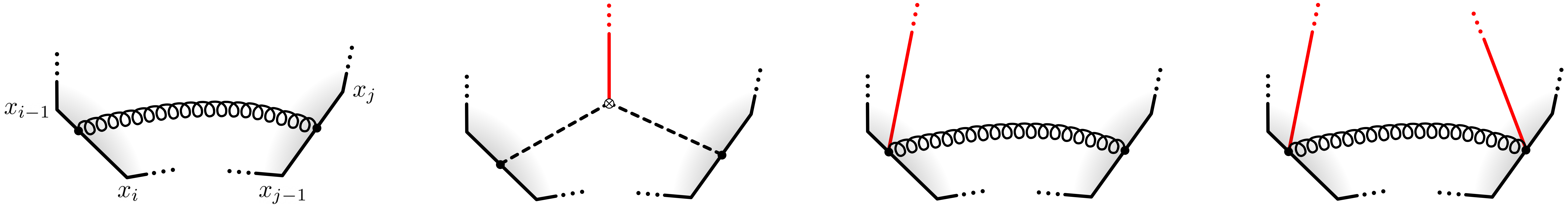




- They generate long-range corrections to the corner vertex $\mathcal{V}_{ii+1} \rightarrow \mathcal{V}_{ij}$
- One-loop result is a sum of R-invariants dressed by transcendentality-2 functions (box integrals)

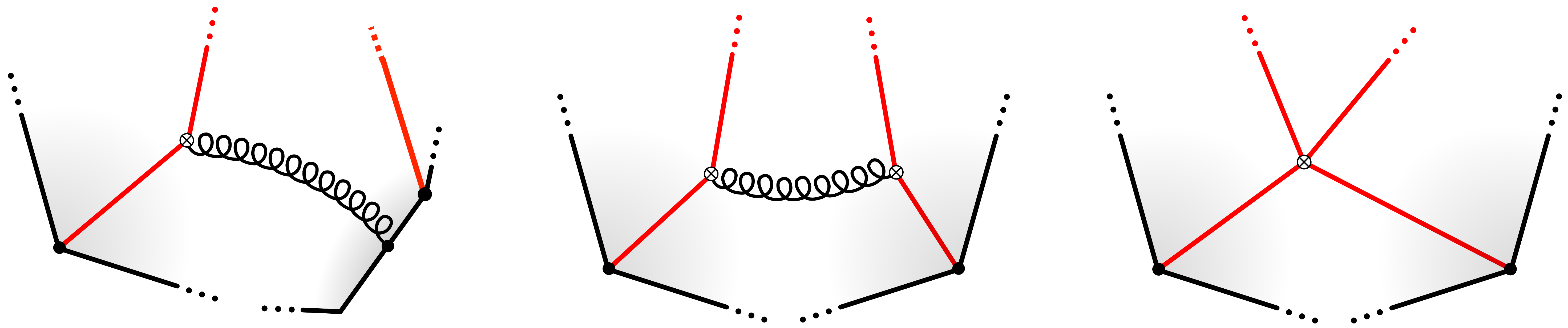
$$W_{k=3,n}^{\text{1-loop}} = \sum_{i,j} \text{tree}(1 \ldots ij \ldots n) \times F_{ij}$$

- Match precisely known form-factor result obtained with generalized unitarity method

[Penante,Spence,Travaglini,Wen]
[Brandhuber,Penante,Travaglini,Wen]

Loop diagrams

- 2nd class diagrams - contain IR divergences



- These divergences are dual to the UV divergences of the local operators
- Half-BPS operators are protected: we observe complete cancellation among these diagrams

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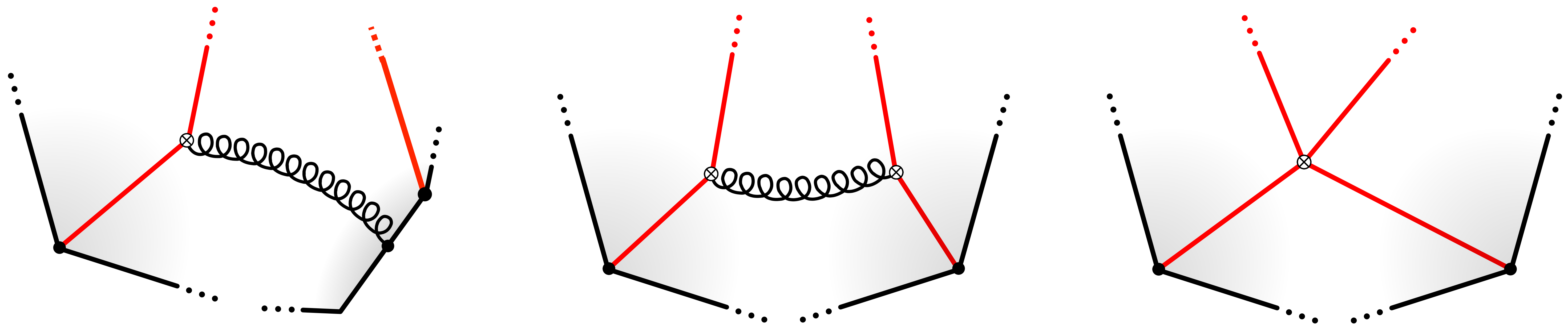
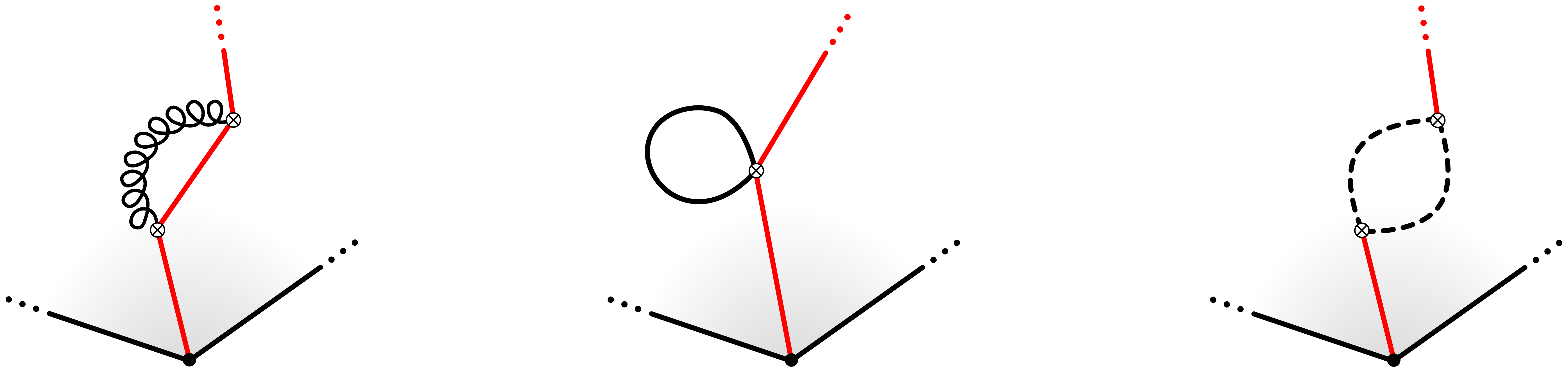
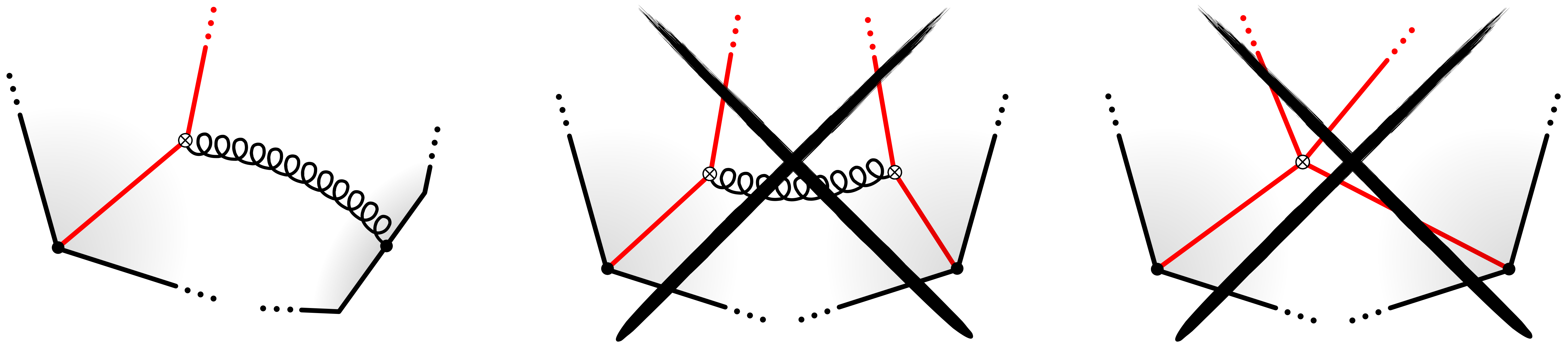


- These divergences are dual to the UV divergences of the local operators
- Half-BPS operators are protected: we observe complete cancellation among these diagrams
- Self-energy diagrams are needed for a complete cancellation

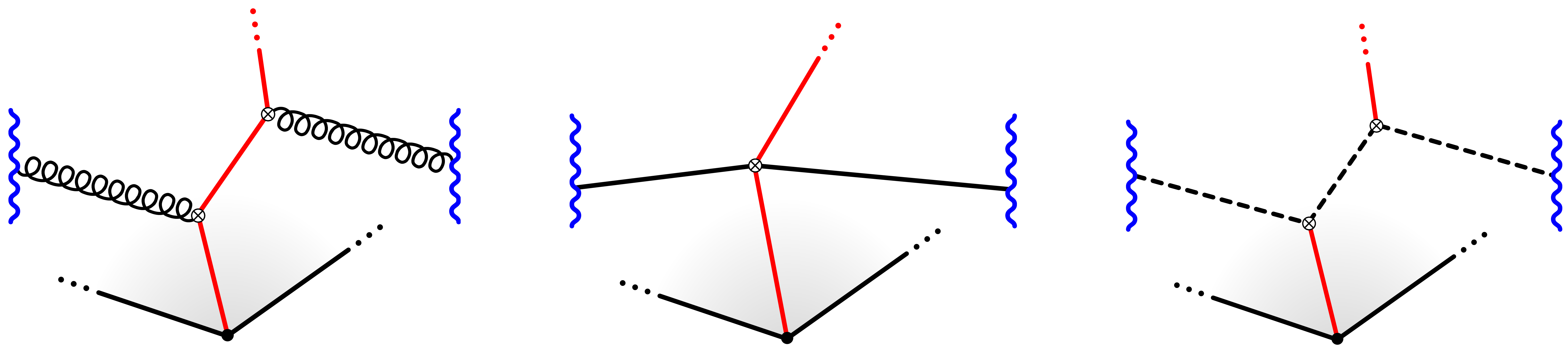


Loop diagrams

- k = 3 (one scalar state) is special



- Fewer diagrams are contributing and cancellation may seem incomplete
- But new diagrams become available at the same time
- Wrapping diagrams with particles winding around the periodic direction are needed

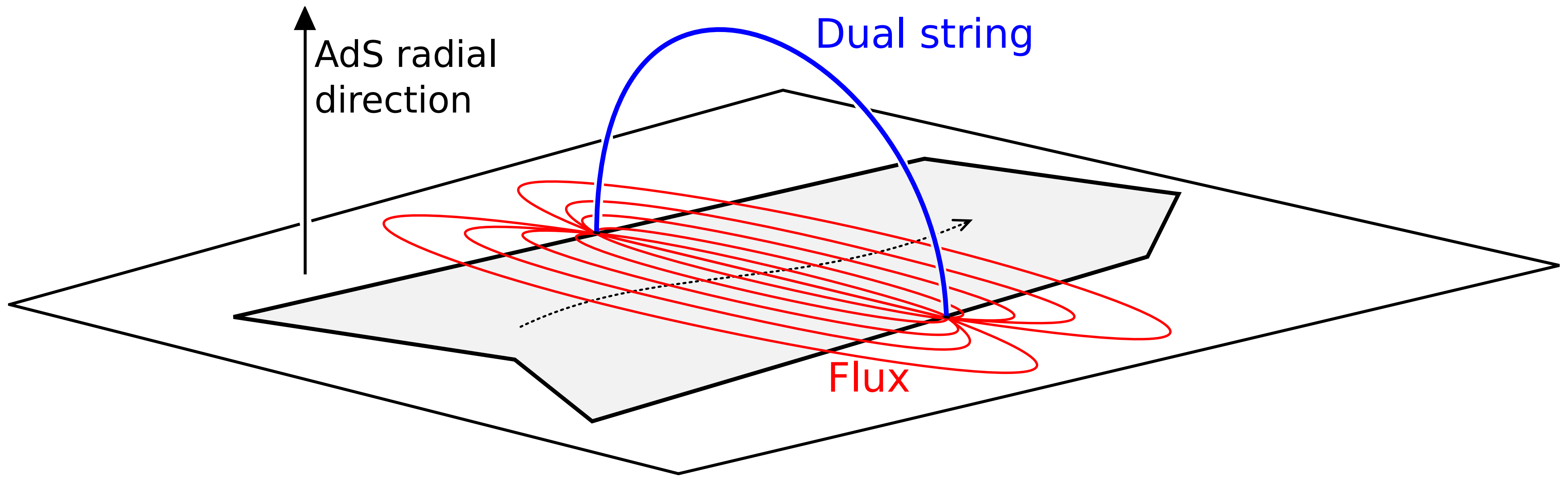


Form Factor OPE and Integrable Bootstrap

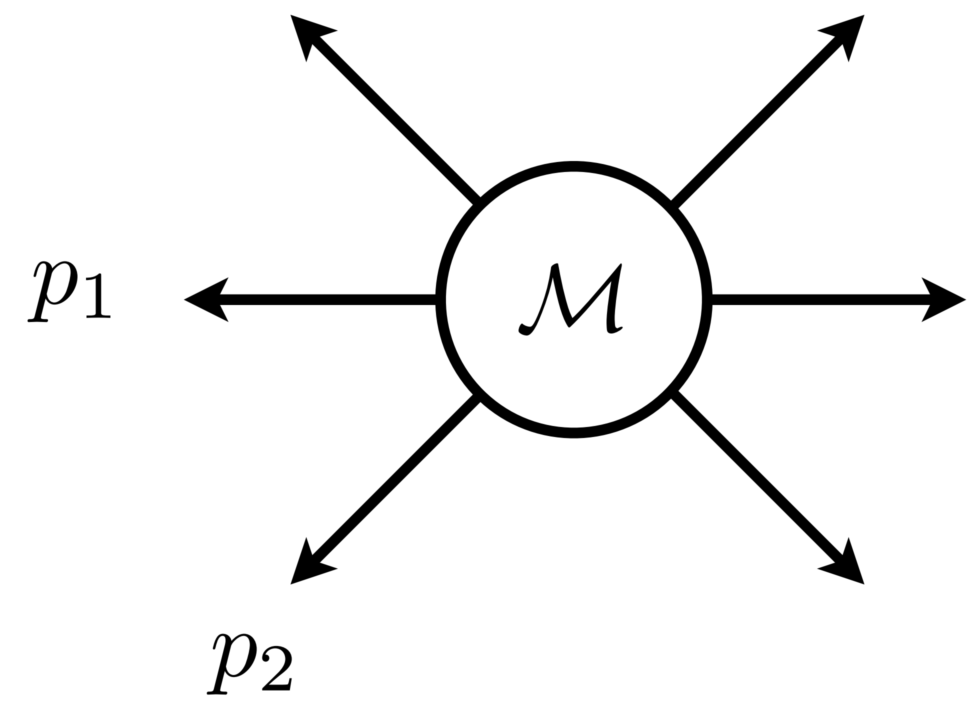
Operator Product Expansion

- Null Wilson loop picture allows us to develop **OPE** for scattering amplitudes

[Alday,Gaiotto,Maldacena,Sever,Vieira]
[BB,Sever,Vieira]



- Idea : Write WL as a sum over a complete basis of eigenstates of the string / flux tube ending on two null edges



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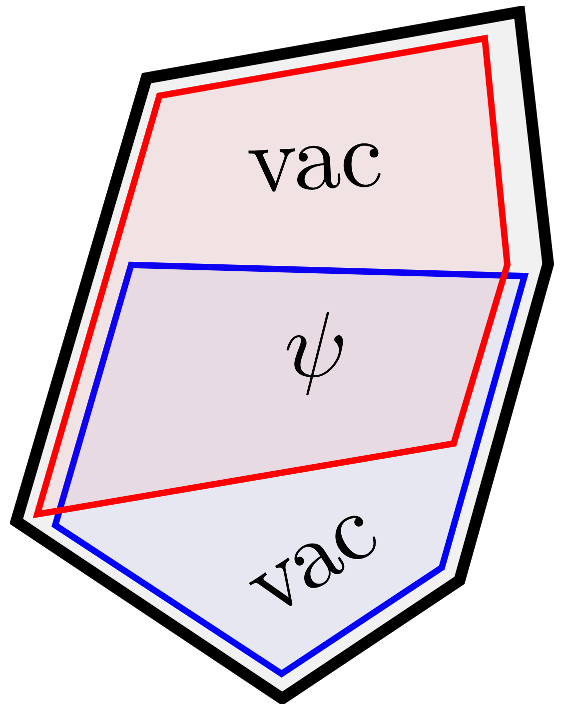


$$= \sum_{\psi} e^{-\tau E(\psi)} P(0|\psi) P(\psi|0)$$

Building blocks:

1. **Spectrum** of flux tube eigenstates

2. **Pentagon transitions** between flux-tube states

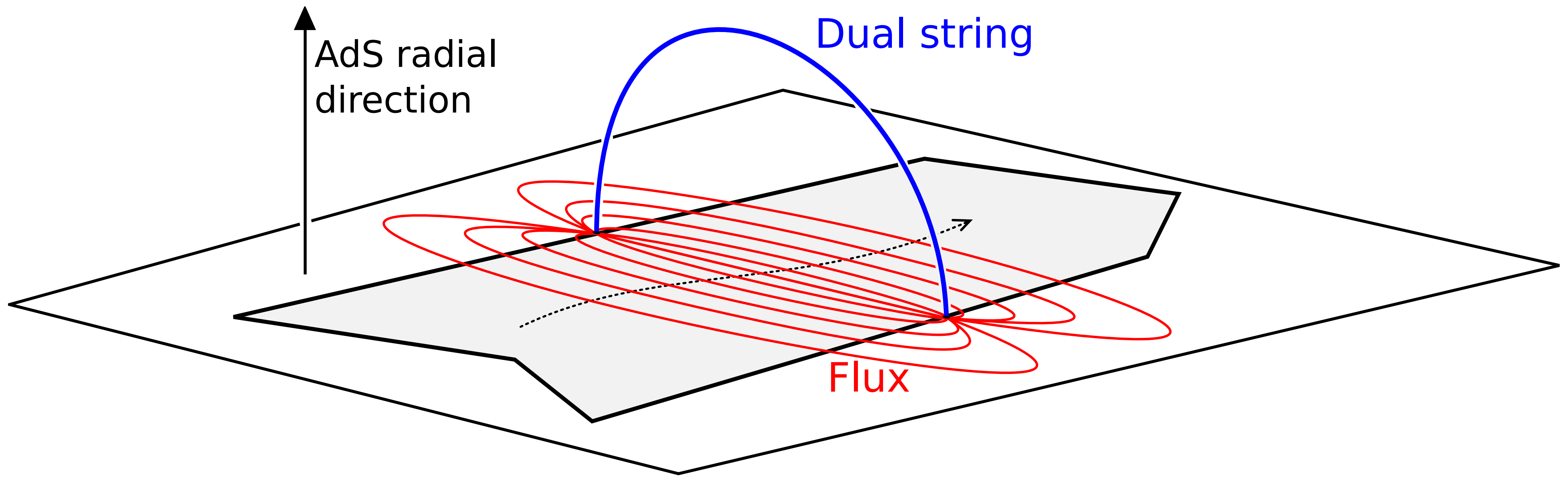
Integrability allows us to determine both data at finite coupling

- The state is produced/absorbed by bottom/top pentagon
- Pentagon transitions are akin to the structure constants for the usual OPE

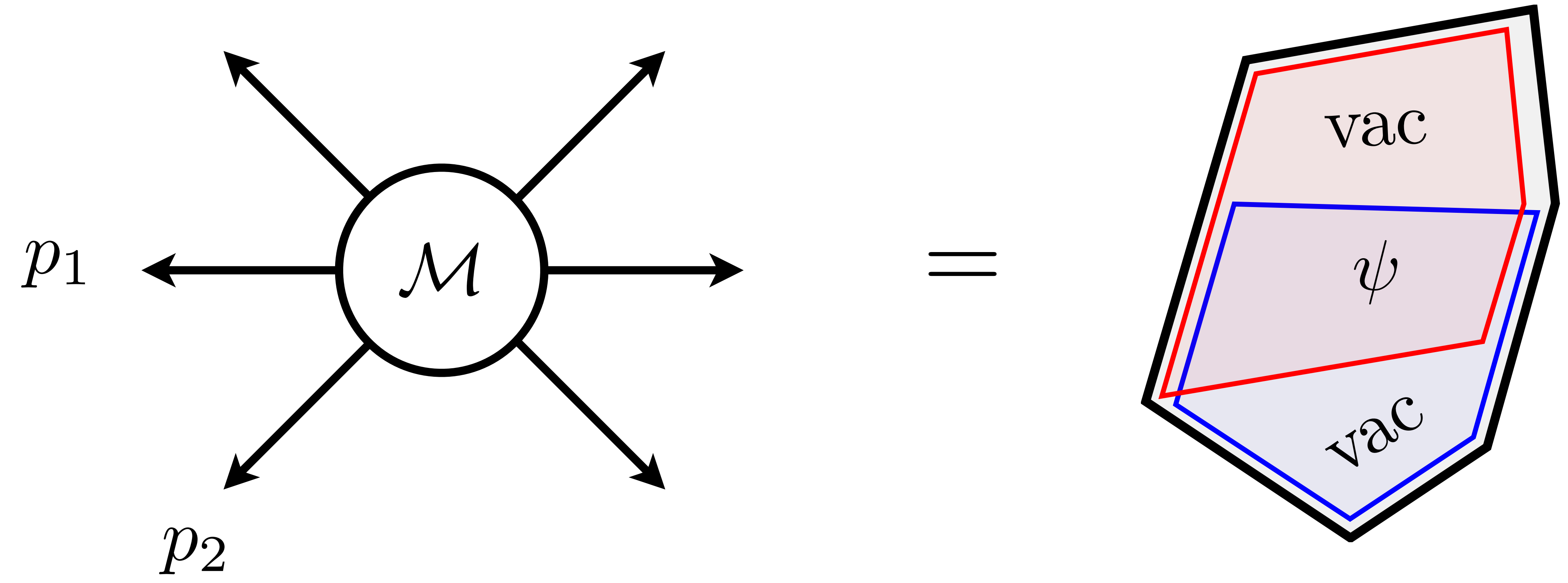
Operator Product Expansion

- Null Wilson loop picture allows us to develop **OPE** for scattering amplitudes

[Alday,Gaiotto,Maldacena,Sever,Vieira]
[BB,Sever,Vieira]



- Idea : Write WL as a sum over a complete basis of eigenstates of the string / flux tube ending on two null edges



$$= \sum_{\psi} e^{-\tau E(\psi)} P(0|\psi) P(\psi|0)$$

Building blocks:

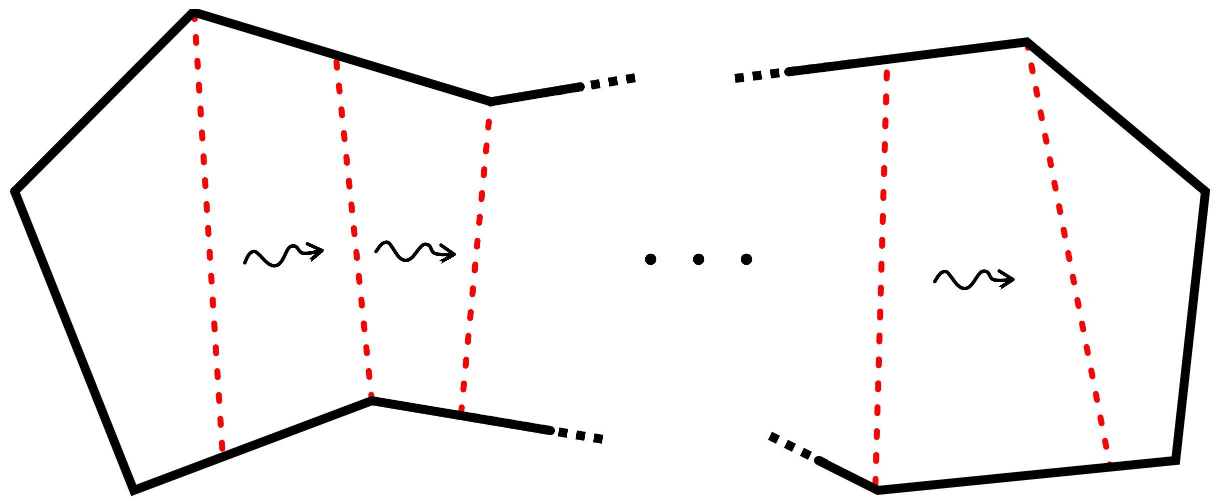
1. **Spectrum** of flux tube eigenstates

2. **Pentagon transitions** between flux-tube states

Integrability allows us to determine both data at finite coupling

- Interpretation: OPE provides a systematic expansion around the collinear limit
- Key advantage: non-diagrammatic - OPE is valid at any value of the coupling constant

Operator Product Expansion

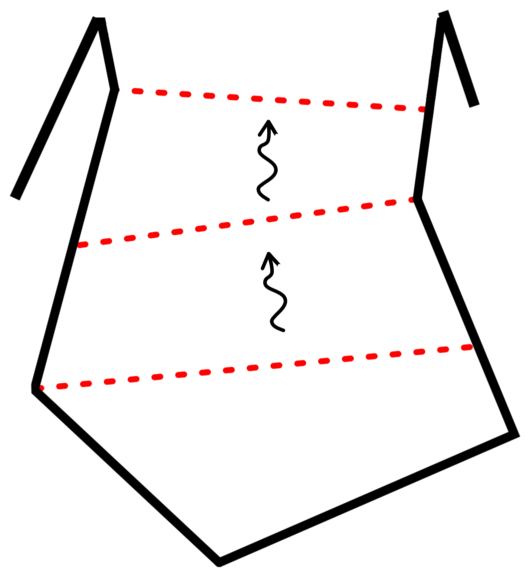


- The more gluons we scatter the more pentagons we need

$$\sum_{\psi_1,\ldots} e^{-\tau_1 E(\psi_1)-\ldots-\tau_{n-5}E(\psi_{n-5})} P(0|\psi_1)P(\psi_1|\psi_2)P(\psi_2|\psi_3)\ldots P(\psi_{n-5}|0)$$

- Form factors may be addressed similarly using periodic Wilson loops
- Main difference: WL produces a state that is contracted with the dual state (~ boundary state)
- Pentagon sequence ends at the top with the so-called **form factor transitions**

[Sever,Tumanov,Wilhelm]

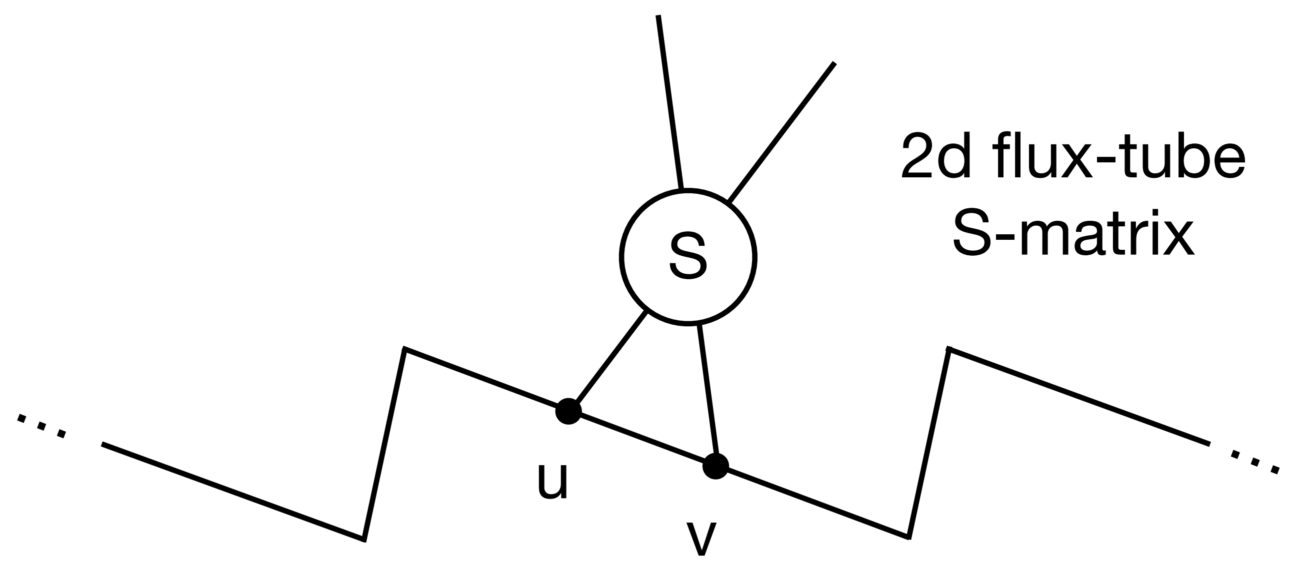
$\langle \mathcal{F}_k |$  $\rangle = \sum_{\psi} e^{-\tau E(\psi)} P(0|\psi)\times \mathcal{F}_k(\psi)$

Amplitude for absorption of the flux-tube state by the half-BPS operator

- FF transitions provide a complete description of form factors in collinear limit

Integrable bootstrap axioms

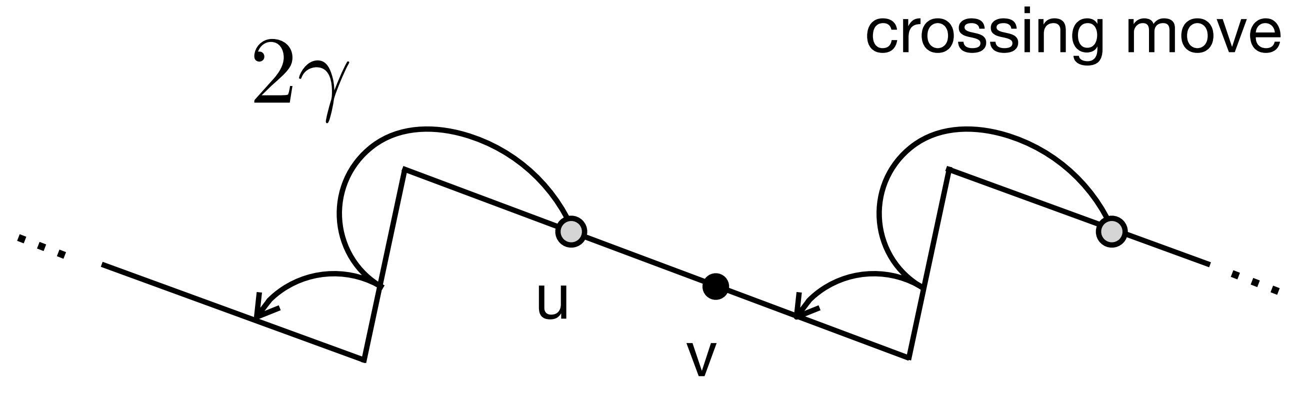
- Two-body form factor transition $F_{\phi\phi}(u,v) = \langle \mathcal{F} | \phi(u)\phi(v)\rangle$



- Main axioms (true at any value of the coupling constant):

- I. Watson equation $F_{\phi\phi}(u,v) = S_{\phi\phi}(u,v) F_{\phi\phi}(v,u)$

- II. Crossing relation $F_{\phi\phi}(u^{2\gamma},v) = F_{\phi\phi}(v,u)$



- III. Kinematical axiom $F_{\phi\phi}(u,u) = 0$ (Fermi exclusion for identical particles)

- Similar axioms were studied for the form factors of the stress tensor multiplet (k=2) [Sever,Tumanov,Wilhelm]

- Difference is that stress tensor produces SU(4) singlets while interested here in symmetric product irreps

BES kernel and its relatives

- Explicit solution for stress tensor makes use of a particular kernel which encodes the coupling constant

[Sever,Tumanov,Wilhelm]

- Kernel belongs to a family of kernels which show up in studies of scattering amplitudes

[BB,Dixon,Papathanasiou]

- *Tilted BES kernel* is defined as a semi-infinite matrix (i, j >1)

$$\mathbb{K}(\alpha) = 2\cos\alpha \begin{bmatrix} \cos\alpha\, \mathbb{K}_{\circ\circ} & \sin\alpha\, \mathbb{K}_{\circ\bullet} \\ \sin\alpha\, \mathbb{K}_{\bullet\circ} & \cos\alpha\, \mathbb{K}_{\bullet\bullet} \end{bmatrix} \qquad \mathbb{K}_{ij} = 2(-1)^{ij+j} \int_0^\infty \frac{dt}{t} \frac{J_i(2gt) J_j(2gt)}{e^t - 1}$$

- BES kernel controlling the cusp anomalous dimension $\mathbb{K}_{\text{BES}} = \mathbb{K}(\alpha = \pi/4)$ [Beisert,Eden,Staudacher]

- "Octagon" kernel is defined similarly $\mathbb{K}_{\text{oct}} = \mathbb{K}(\alpha = 0)$

[Coronado]
[Belitsky,Korchemsky]
[Kostov,Petkova,Serban]

- This kernel also plays a role in study of correlation functions of large charge operators

Solution at finite coupling

- Relevance of these kernels is that they allow us to define quantities obeying the axioms we are interested in
- A “general solution” may be written for any value of the deformation parameter (tilt angle) [Sever,Tumanov,Wilhelm] [BB,Tumanov]
- Schematically

$$P^{[\alpha]}(u|v) = \frac{\Gamma(iu - iv)}{g^2\Gamma(\frac{1}{2} + iu)\Gamma(\frac{1}{2} - iv)} e^{-\kappa_\alpha(u)\cdot[1+\mathbb{K}(\alpha)]^{-1}\cdot\kappa_\alpha(v)}$$

- It is designed in a way such that a number of properties are immediately obeyed (such as the Watson eq.)
- The crossing axiom depends however sensitively on the choice of the deformation parameter
- The crossing axiom relevant for the pentagon (closed) Wilson loop requires alpha = pi/4 [BB,Sever,Vieira]

$$P(u|v) = P^{[\alpha=\pi/4]}(u|v)$$

- Its pole at u = v defines a natural 1-pt function / integration measure $\mu(u) = \frac{i}{\text{Res}_{v=u} P(u|v)}$

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- It is designed in a way such that a number of properties are immediately obeyed (such as the Watson eq.)
- The crossing axiom depends however sensitively on the choice of the deformation parameter
- The crossing axiom relevant for the zig-zag (periodic) Wilson loop requires alpha = 0

$$Q(u|v) = P^{[\alpha=0]}(u|v)$$

- Its pole at u = v defines a new 1-pt function / integration measure

$$\nu(u) = \frac{i}{\mathrm{Res}_{v=u} Q(u|v)}$$

General solution

- General (factorized) solution for absorption of k-2 scalars by the half-BPS operator $\mathrm{Tr}\,\phi^k$

$$\langle \mathcal{F}_k | \phi(u_1), \ldots, \phi(u_{k-2}) \rangle = \prod_{i<j}^{k-2} \frac{1}{Q(u_j|u_i)}$$

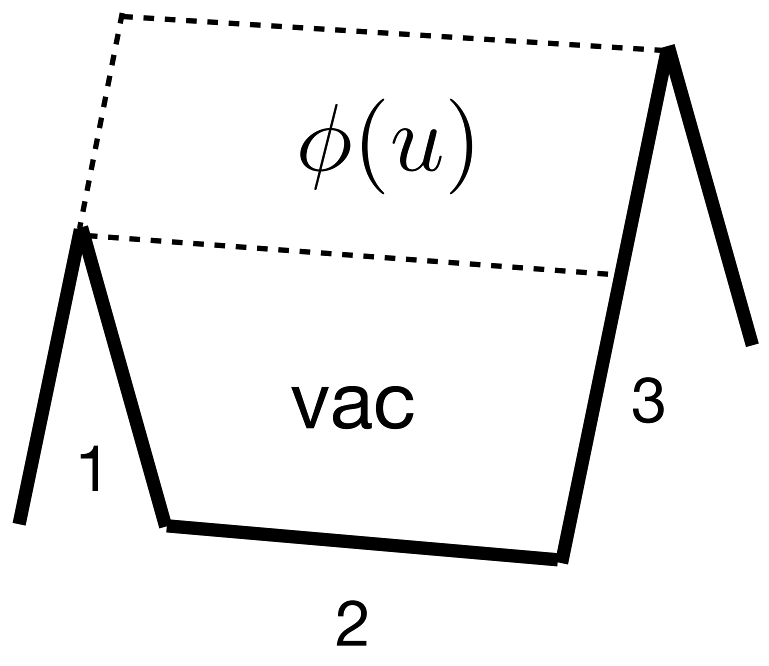
- It may be shown to verify all the bootstrap axioms
- All the complicated coupling constant dependence resides in the two-body form factor
- 1pt form factor is absorbed in the measure used to integrate over the (k-2)-scalar phase space

$$\int \mathrm{dPS} = \frac{1}{(k-2)!} \int \frac{du_1 \ldots du_{k-2}}{(2\pi)^{k-2}} \prod_{i=1}^{k-2} \sqrt{\mu(u_i)\nu(u_i)}$$

- It combines the two measures derived from the pentagon P and the “octagon” Q

Simplest example

- Exact representation for the collinear limit of the 3pt form factor of the k=3 operator



$$W_{3,3}(u_1,u_2,u_3) = \int \frac{du}{2\pi} \sqrt{\mu(u)\nu(u)}\; e^{i\sigma p(u) - \tau E(u)} + \ldots$$

- Dots stand for subleading terms in the collinear limit $u_3 \approx e^{-2\tau} \to 0$

- The OPE time and space variables parametrize the Mandelstam invariants

$$u_3 = \frac{s_{12}}{q^2} = \frac{1}{1+e^{2\tau}} \qquad u_1 = \frac{s_{23}}{q^2} = \frac{1}{1+e^{2\sigma}+e^{-2\tau}} \qquad u_2 = \frac{s_{31}}{q^2} = \frac{e^{2\sigma}}{(1+e^{2\tau})(1+e^{2\sigma}+e^{-2\tau})}$$

- All ingredients (energy, momentum, measures) are known to all loops

- They may be used to bootstrap the 3pt form factor through 6 loops

[BB,Dixon,Tumanov; in preparation]

Higher charge example

- Exact representation may also be obtained for higher-charge operators. E.g. 4pt k=4

$$W_{4,4} = \int \frac{du\, du_1 du_2}{2(2\pi)^2} \sqrt{\mu(\mathbf{u})\nu(\mathbf{u})} e^{i\sigma_1 p(u) + i\sigma_2 p(u_{1,2}) - \tau_1 E(u) - \tau_2 E(u_{1,2})} \frac{P(u|v_1)P(u|v_2)}{P(v_1|v_2)Q(v_2|v_1)} + \dots$$

- They depend on more kinematic invariants …
- … and result in more integrals / pentagons / flux-tube particles
- Arguably harder to study but here again ingredients are known at any value of the coupling constant
- They may used to put constraints on form factors of higher-charge operators at higher loops

Summary

- Form factors of half-BPS ops admit a dual description in terms of null Wilson loops
- Unlike for scattering amplitudes the Wilson loops here are infinite and periodic
- Local operators are expected to map to on-shell states in the dual picture
- Most natural choice for half-BPS operators are states made out of zero-momentum scalars
- This picture may be checked explicitly through one loop at weak coupling
- It may also be used to motivate and develop the OPE approach for calculating form factors at finite coupling
- Solution may be found to all loops using integrability building blocks such as the tilted BES kernel

Outlook

- Can we use Wilson loop picture to calculate non-MHV form factors?
- This may naively be done by considering more general super Wilson loop operators - would be nice to check it
- Form factors of non-protected operators?
- Any local operator should admit a dual description
- Precise dictionary is still largely unknown
- One may hope to make progress for simplest non-protected operator: Konishi operator $\mathcal{K} = \epsilon_{ABCD} \operatorname{Tr} \phi^{AB} \phi^{CD}$
- Its form suggests to look for a dual R-singlet state
- Can we make sense of it? Can we bootstrap its form factors using integrability?
- May bridge the gap between integrable structures governing spectral problem and scattering amplitudes

Thanks!