Fracton States of Matter as Toy Models of Holography

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+ work in progress with A.Prem (Princeton) and A.Thomas (CU Boulder)

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The Question

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- ☑ in the informational aspect (entanglement structure)?

Outline

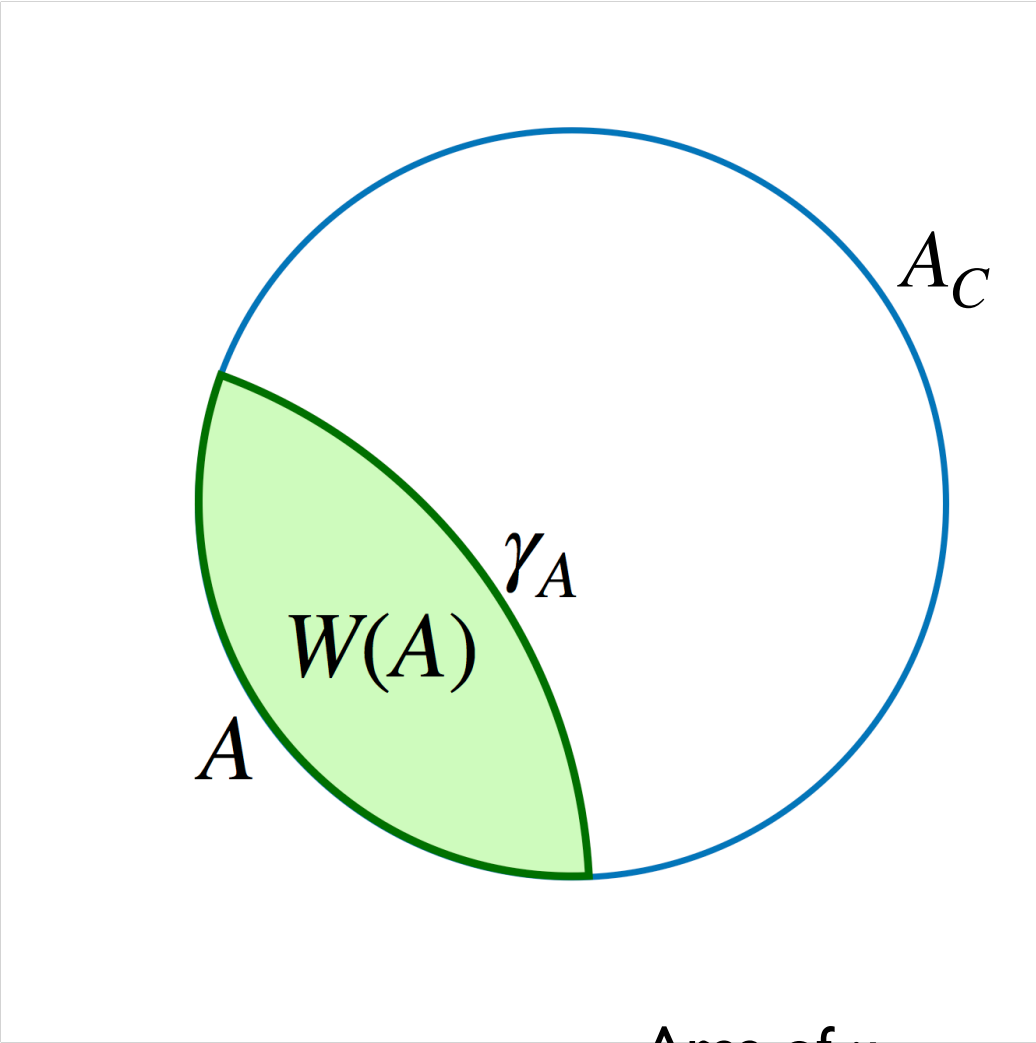
I. **Brief introduction of holography**

II. The classical fracton model on flat and hyperbolic lattices

III. Holographic properties of the hyperbolic fracton model

IV. Unifying different holographic toy models by Lifshitz gravity/symmetric rank-2 U(1) gauge theory

Ryu-Takayanagi Formula



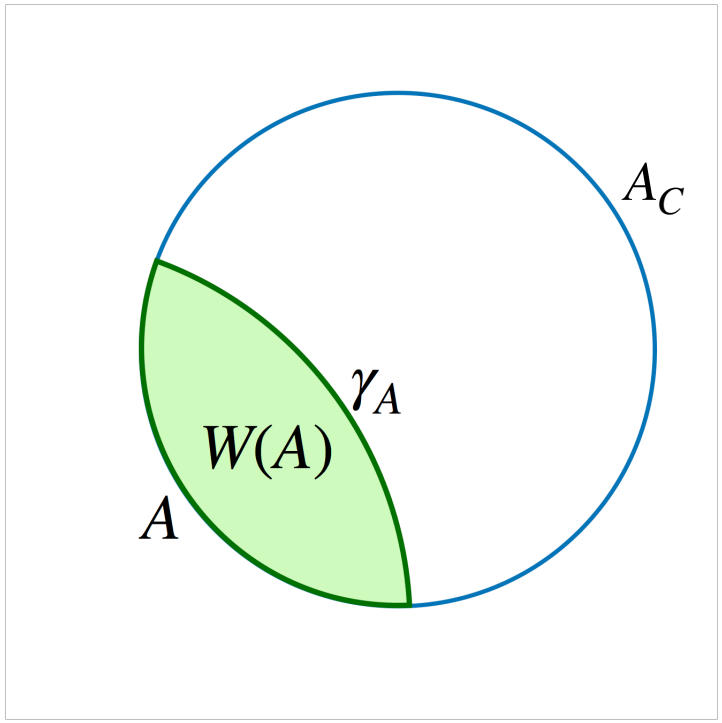
Entanglement between A & A_c

$$S_A = \frac{\text{Area of } \gamma_A}{4G_N} (+\text{bulk corrections})$$

[Ryu, Takayanagi]

Rindler Reconstruction

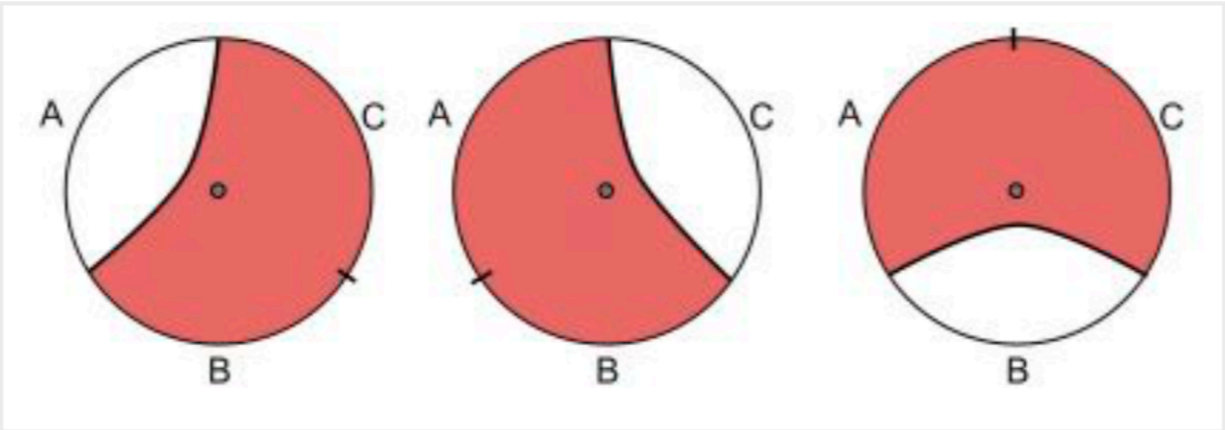
AdS-Rindler reconstruction:
Given information on boundary region A,
bulk information in the **geodesic wedge W(A)** can be reconstructed;
bulk information beyond the **geodesic wedge W(A)** cannot.



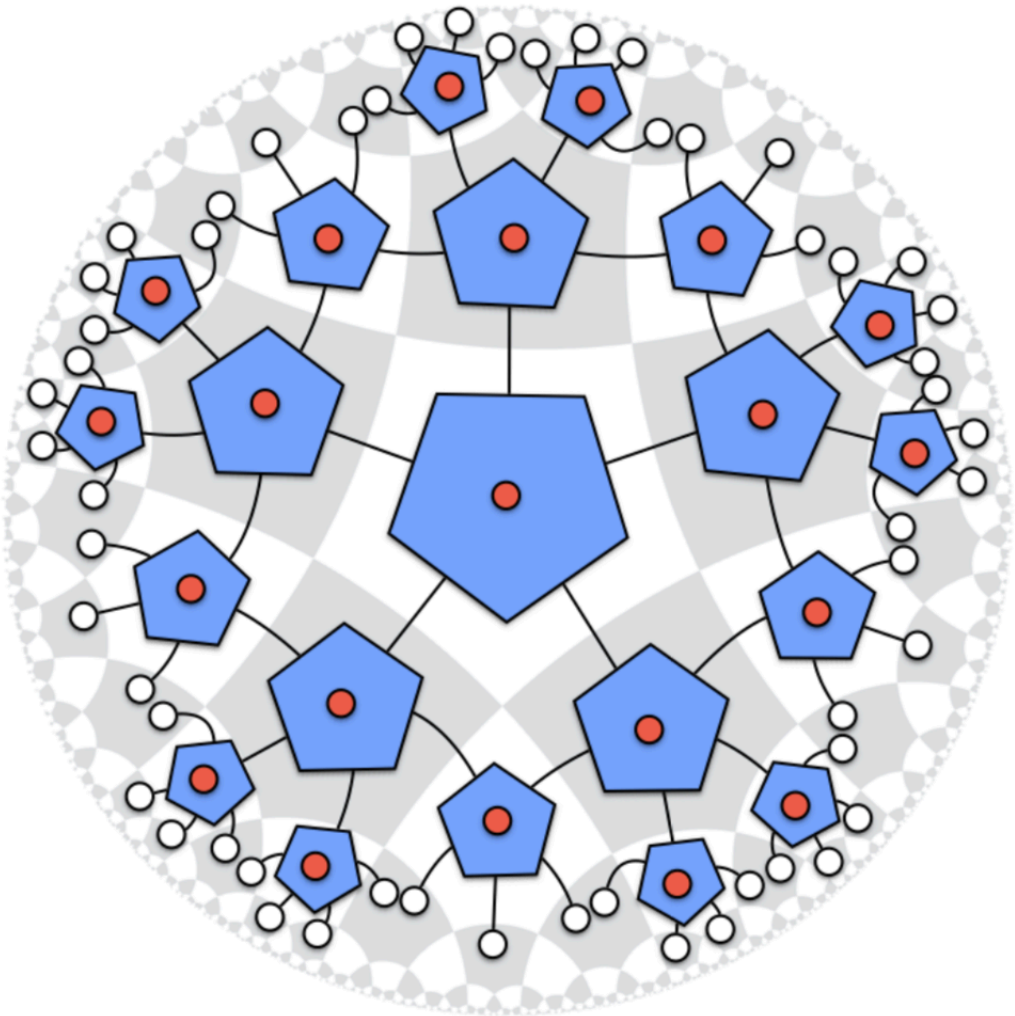
Rindler Reconstruction

Non-trivial encoding:

Center bulk op. is in the geodesic wedge of AB, BC, CA but not the wedge of A, B, or C individually



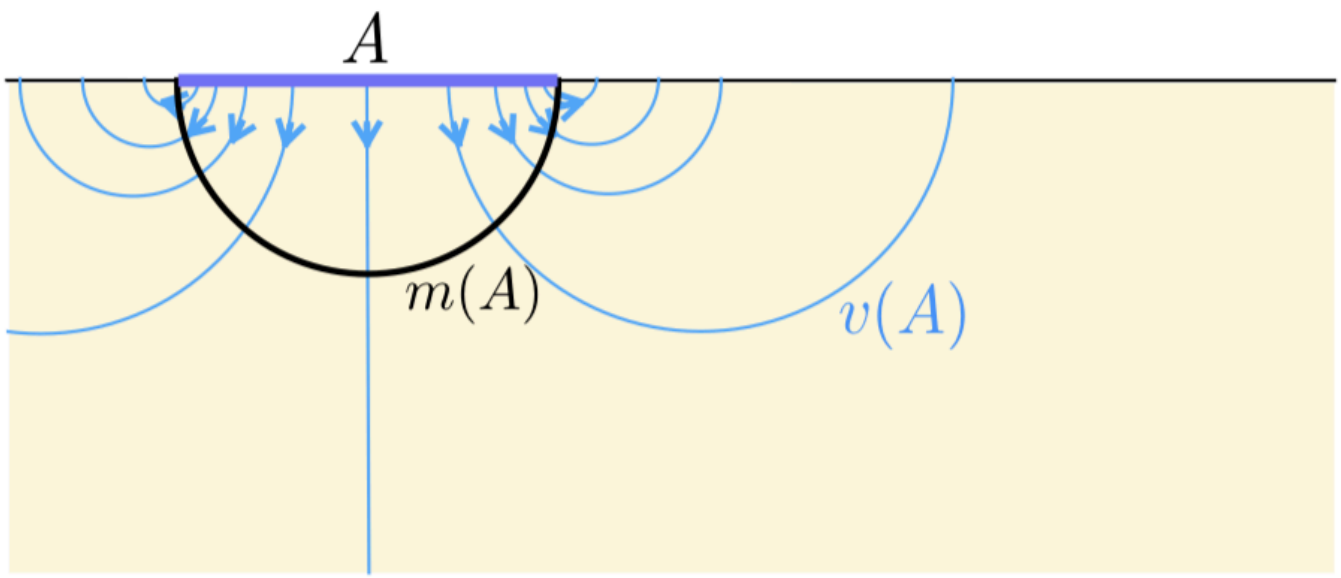
Known Toy Models



Perfect tensor networks and random tensor networks

[HaPPY (Harlow, Preskill, Pastawski, Yoshida), Qi, …]

Known Toy Models



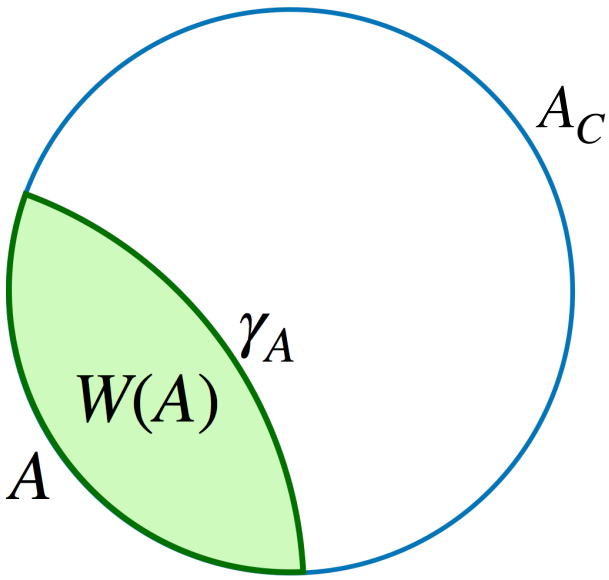
Bit-thread constructions

[Freedman, Headrick, Wen, …]

What we will show…

A classical many-body toy model (fracton states) mimics the exotic properties of gravity —

- [x] in the field theory aspect (gravitons)?
- [✓] in the informational aspect (entanglement structure)?



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I. Brief introduction of holography

II. The classical fracton model on flat and hyperbolic lattices

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IV. Unifying different holographic toy models by Lifshitz gravity/symmetric rank-2 U(1) gauge theory

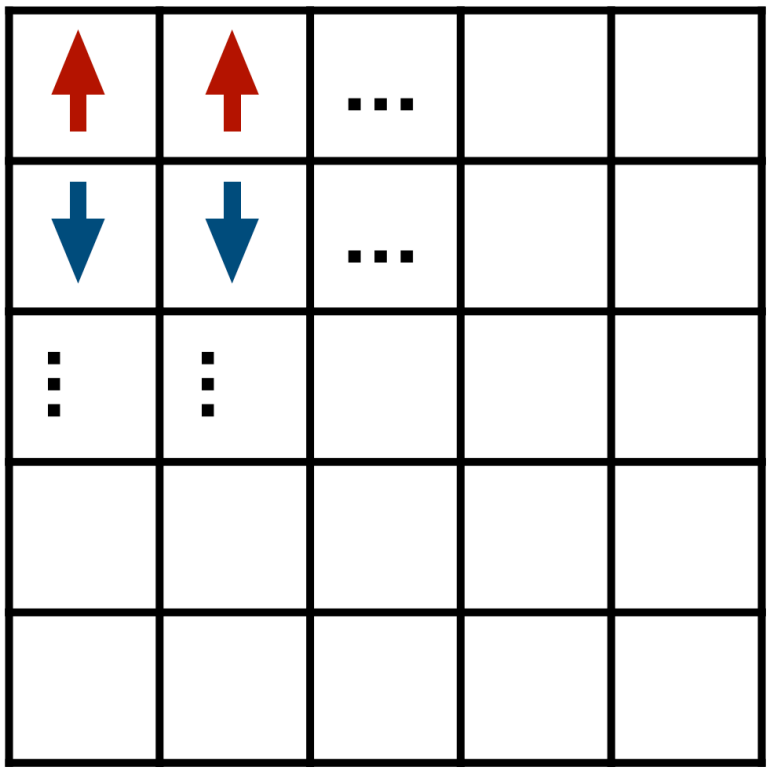
[Chen, Slagle, Shirley, Xu, Fu, Vijay, Balents, Pretko, Haah, Halasz, Hsieh, Ma, Kim…]

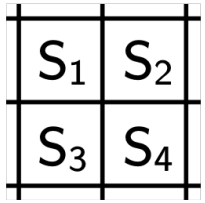
A BIT OF HISTORY

1. Gauge theory with symmetric rank-2 U(1) electric fields
   [Xu, …]
2. R2-U1 found to be some versions of Lifshitz gravity
   [Xu, Horava …]
3. Fracton topological order was discovered
   [Fu, Vijay, Haah,…]
4. R2-U1 + Higgs = gapped Fracton topological order / other fracton states, featuring subsystem symmetries
   [Chen, Slagle, Pretko, Ma, …]

Fracton Model on a Flat Lattice

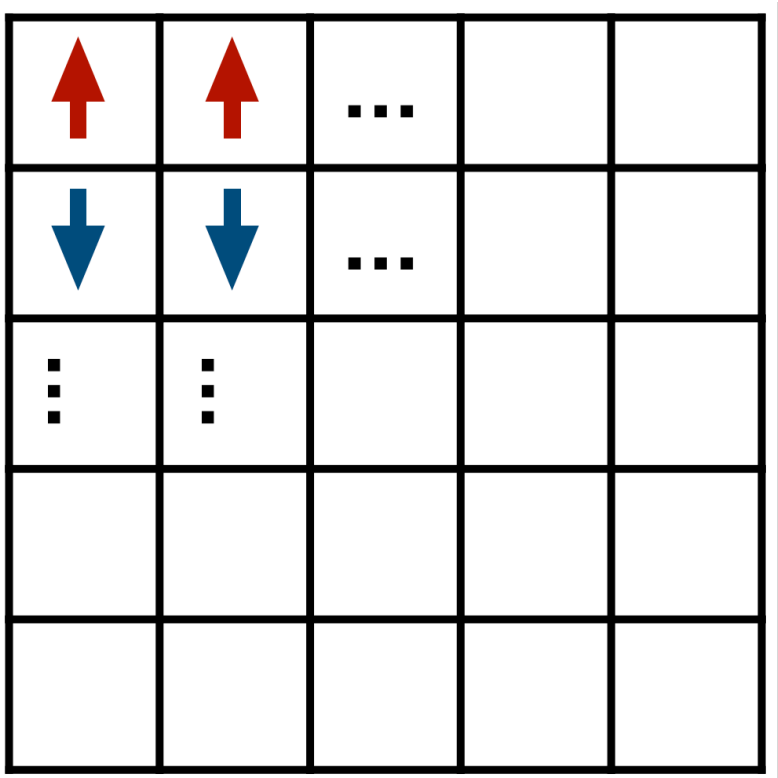
Sites on the center of square plaquette,

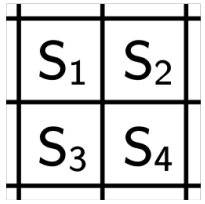




$$\mathcal{O}_p = \prod_{i=1}^{4} S_i^z$$

Fracton Model on a Flat Lattice



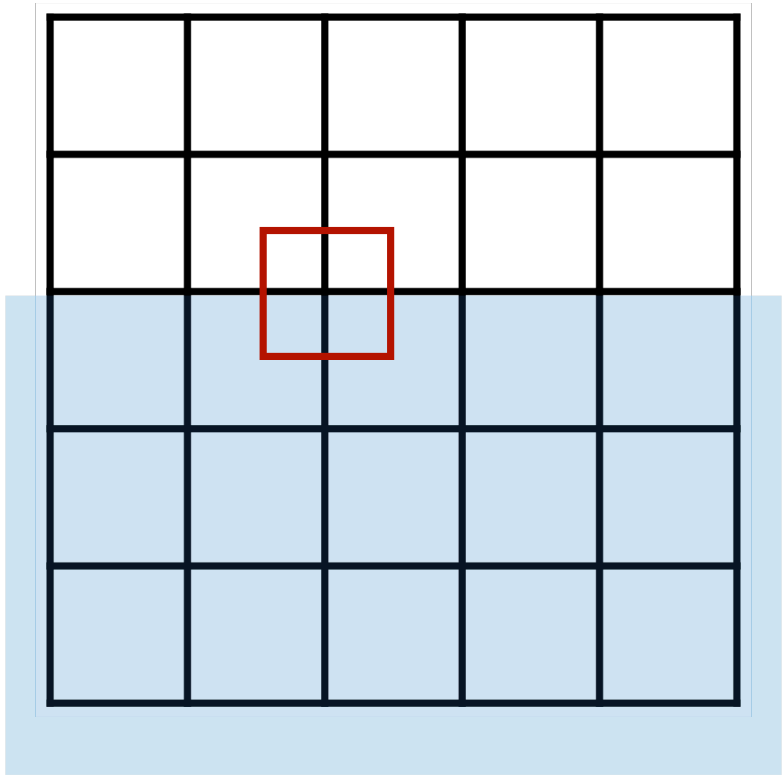


$$\mathcal{O}_p = \prod_{i=1}^{4} S_i^z$$

Ground state: $O_p=1$ for all four-spin clusters

Ground state degeneracy: $\sim 2^{Lx+Ly}$

Fracton Model on a Flat Lattice



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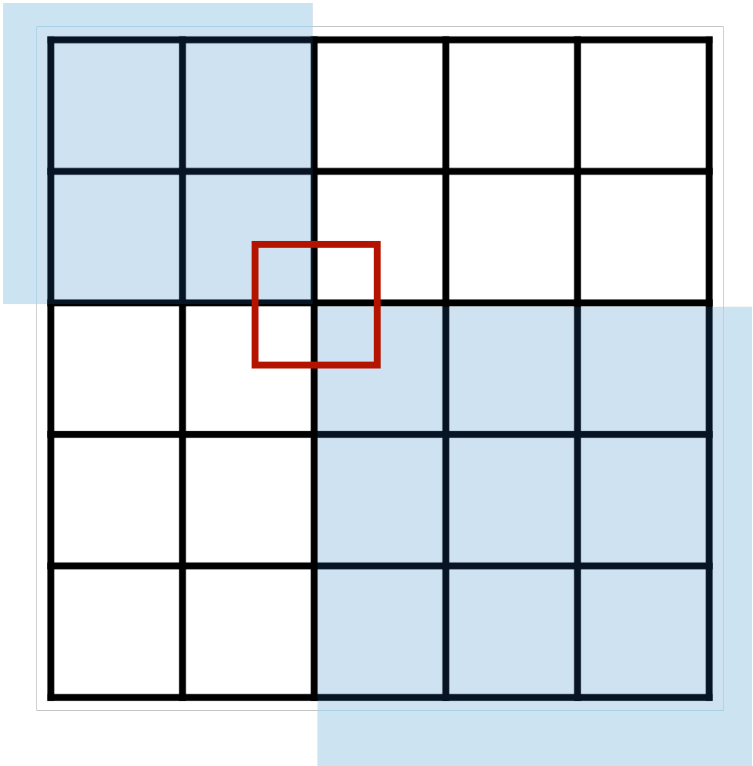

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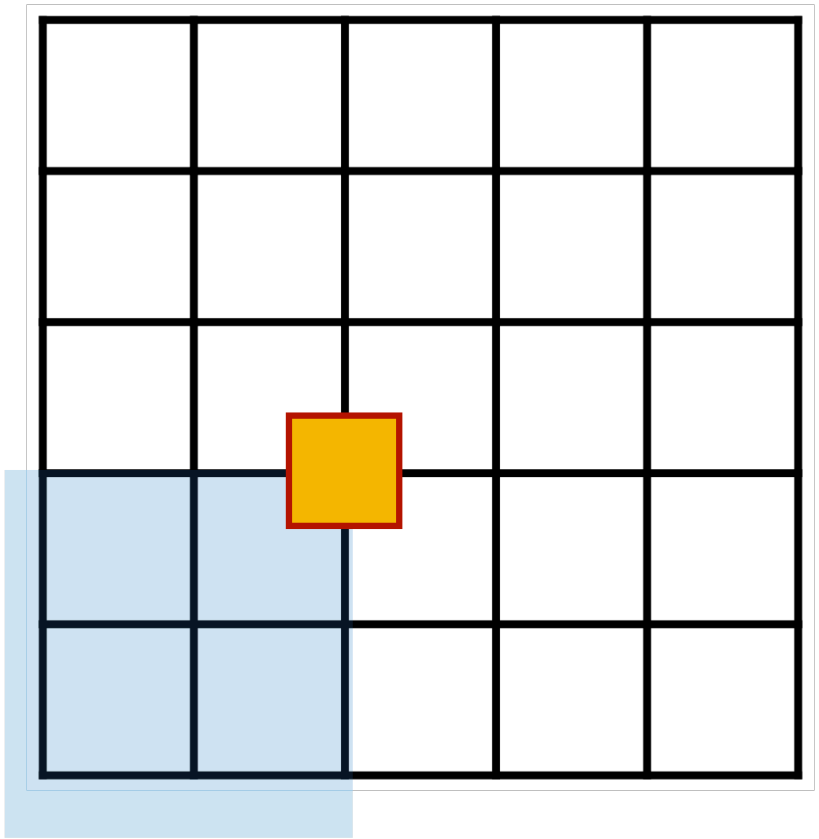
Ground state degeneracy: $\sim 2^{Lx+Ly}$

Fracton Model on a Flat Lattice

First excited state: a fracton
O_p=-*1* for one plaquette
O_p=*1* for all other plaquettes

Cannot be created by local operation (without extra energy cost)

Cannot be moved by local operation (without extra energy cost)

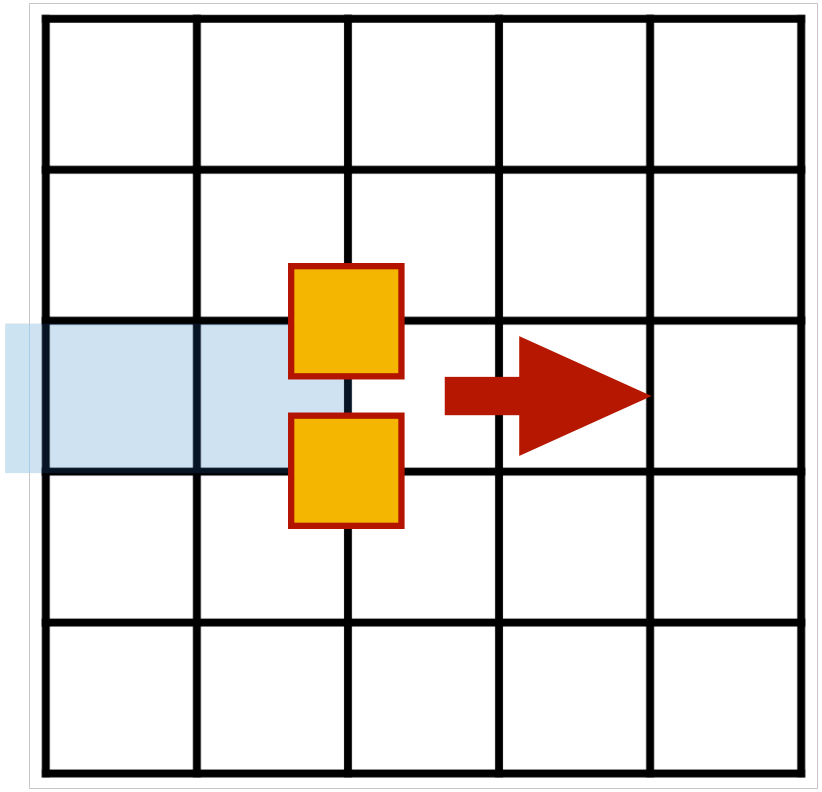


Fracton Model on a Flat Lattice

Bound states of fractons:

2-fracton bound state:
can move in 1D

4-fracton bound state:
can more freely



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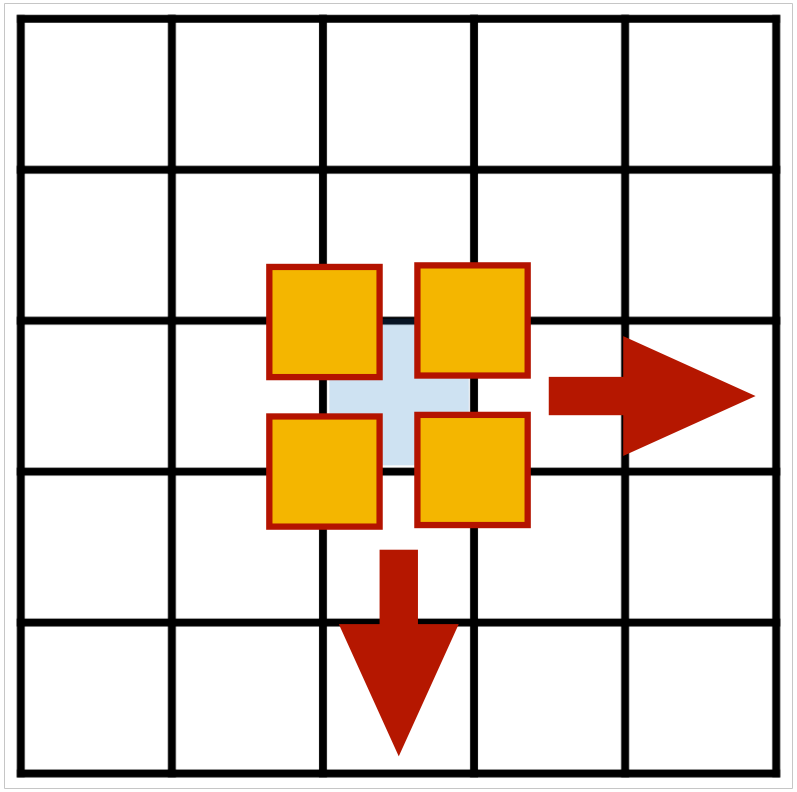
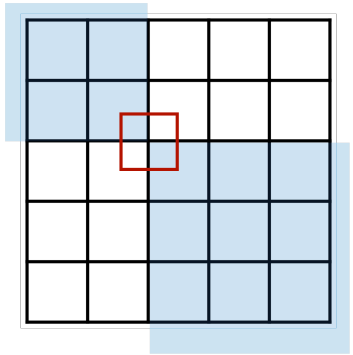
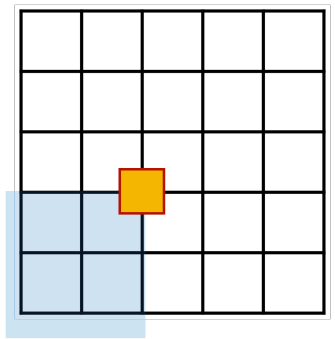


Fracton Model on a Flat Lattice

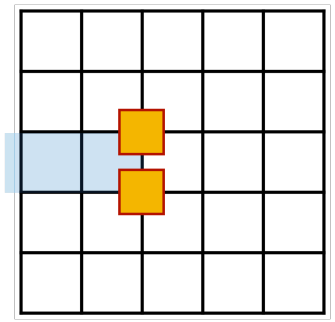
Sub-extensive ground state degeneracy (subsystem symmetry)

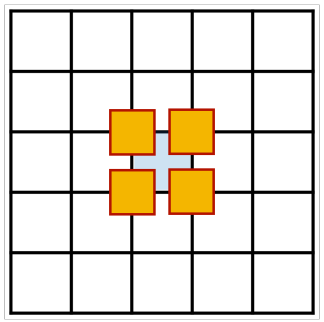


Fracton excitations can only move in a sub-manifold of the system



Bound states of fractons can move in higher dimensions



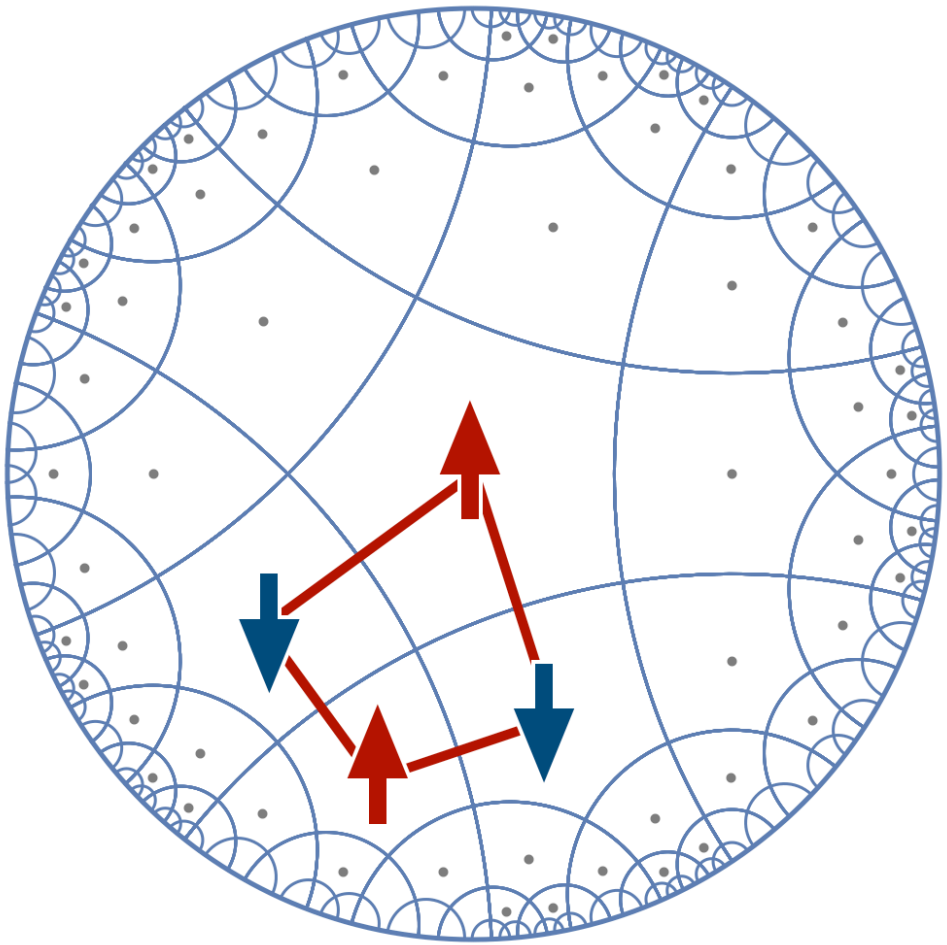


The Model

Sites on the center of pentagon form a square lattice

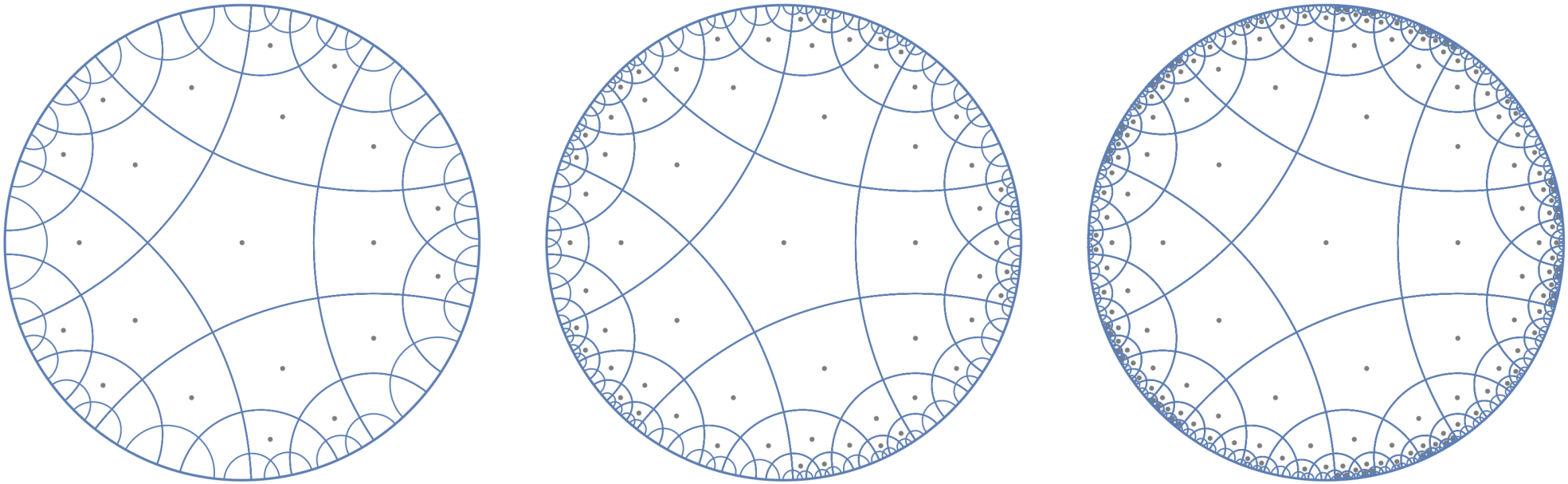
$$\mathcal{O}_p = \prod_{i=1}^{4} S_i^z$$

$$H = -\sum_p \mathcal{O}_p$$



Geodesics (straight lines) are edges of pentagons

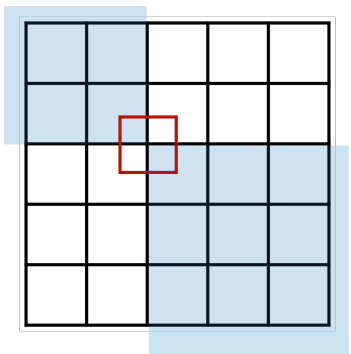
The Model

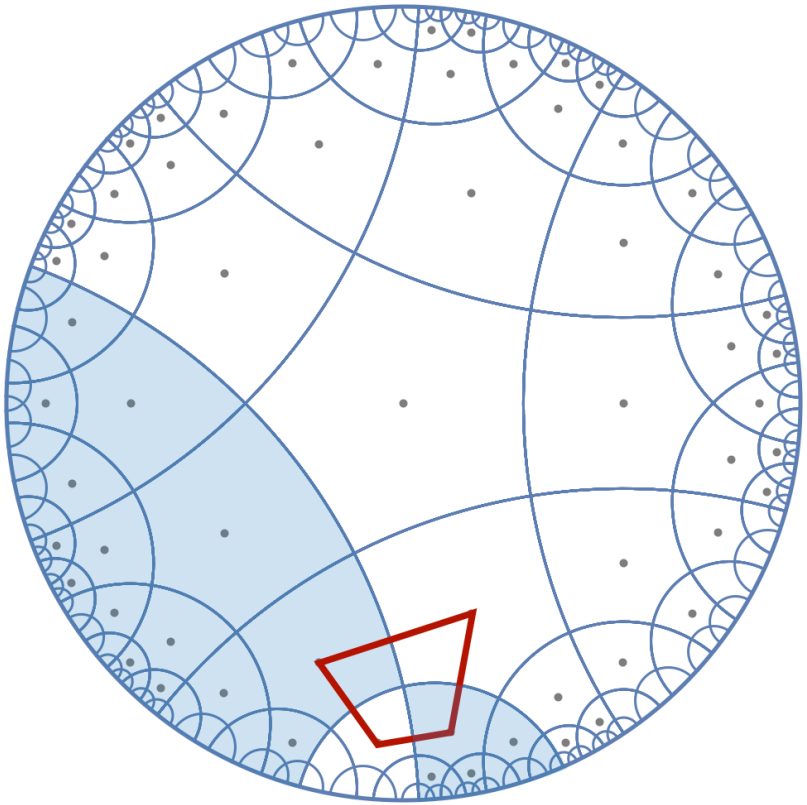


Lattice of different size: characterized by number of geodesics N_g

Ground State Degeneracy

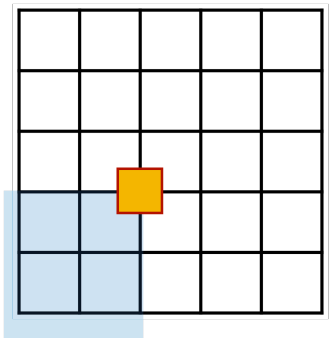
Ground state:
Flip an entire geodesic wedge.
Degeneracy: 2^{N_g+1}

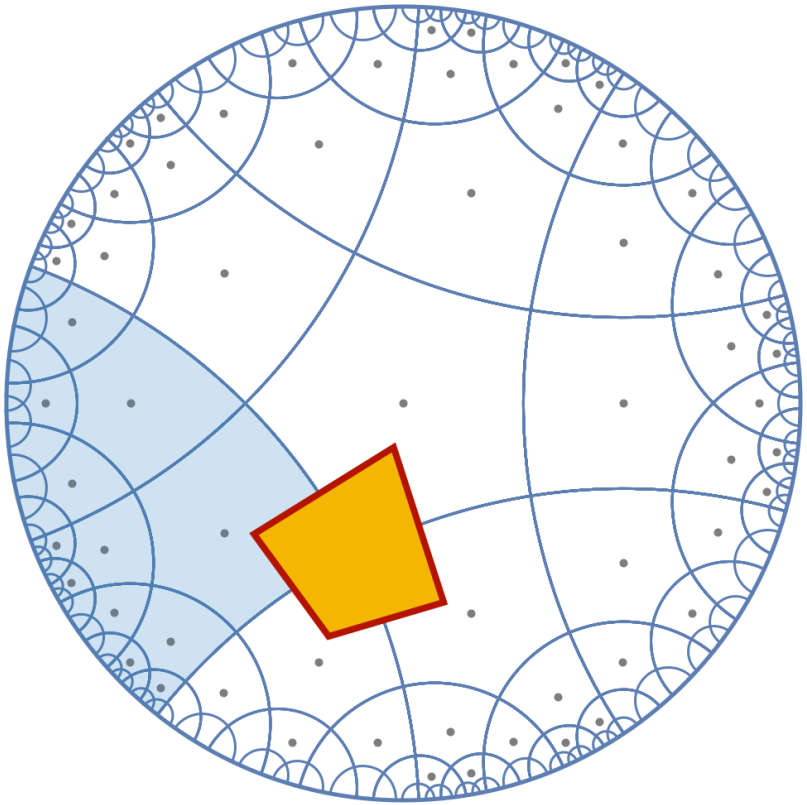




Fracton Excitation

1-fracton state:
Flip a region covered by
intersecting geodesics





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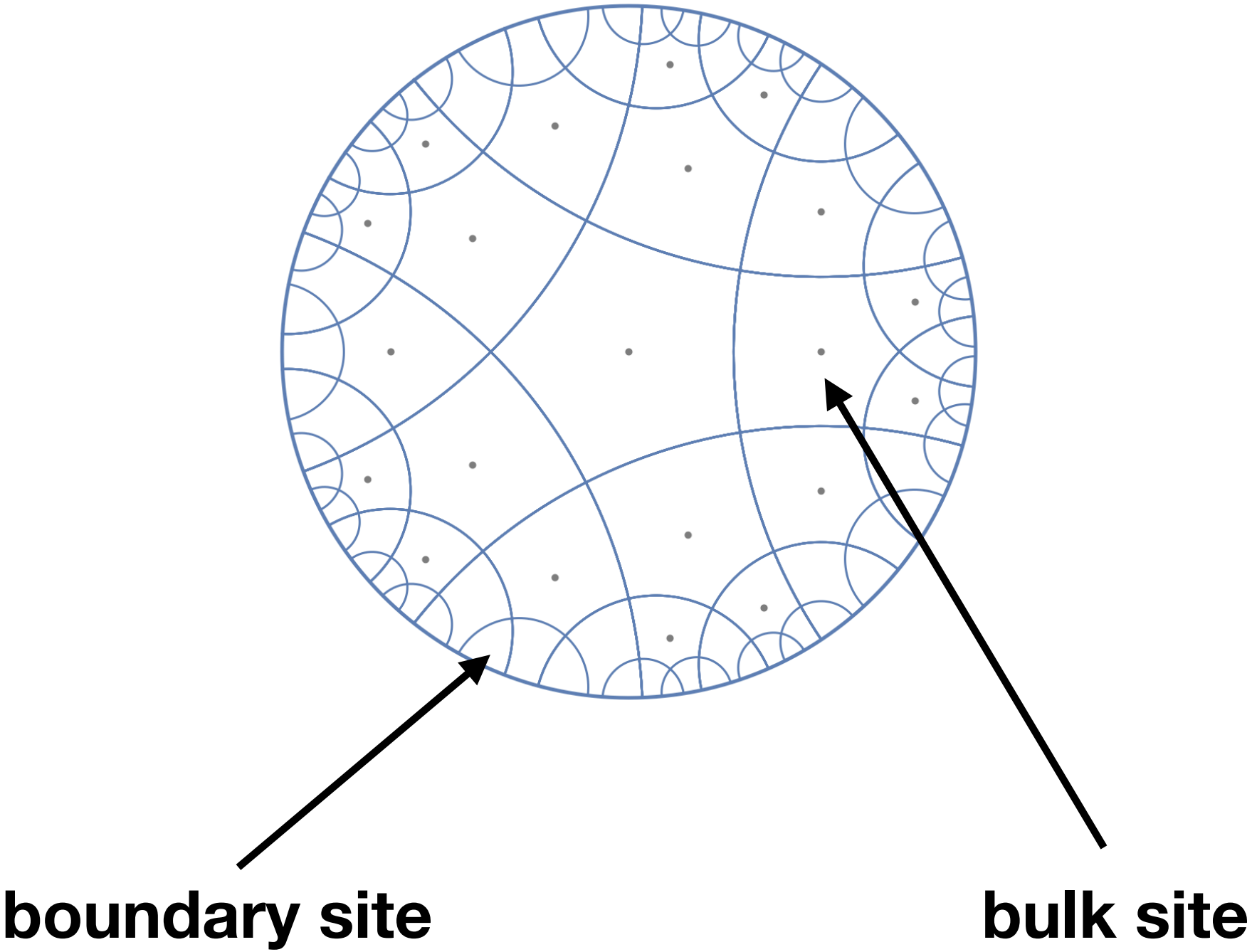
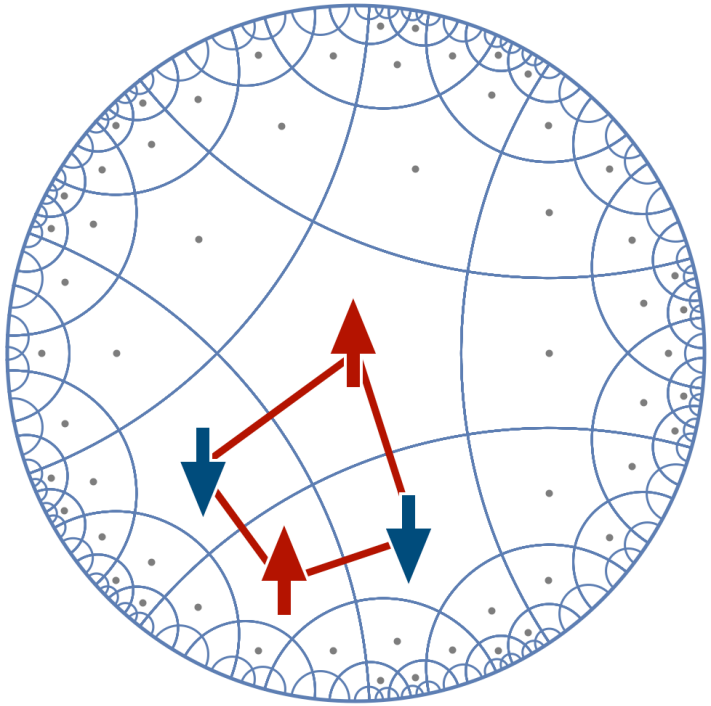
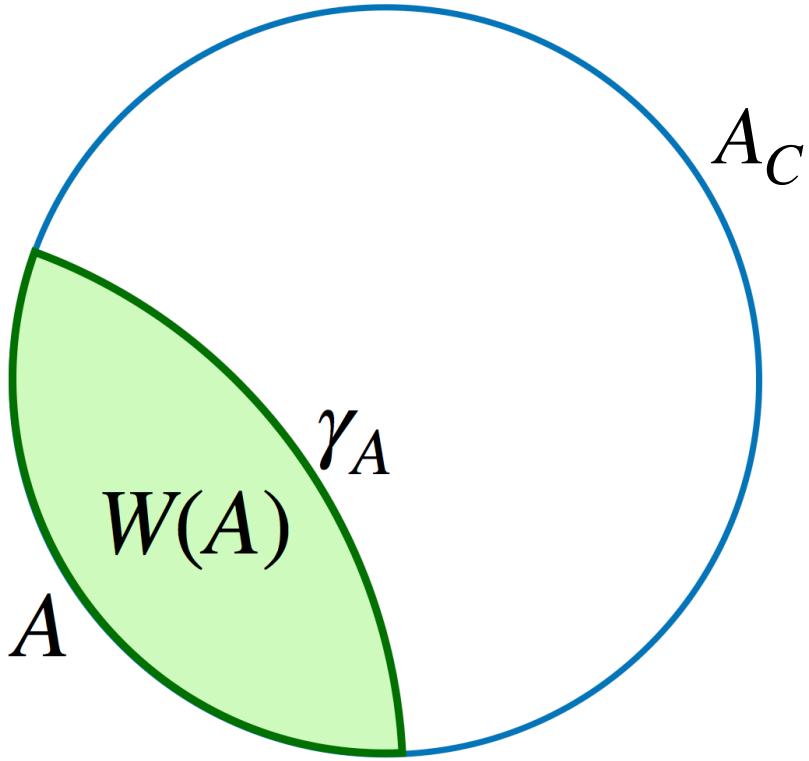

boundary site
bulk site

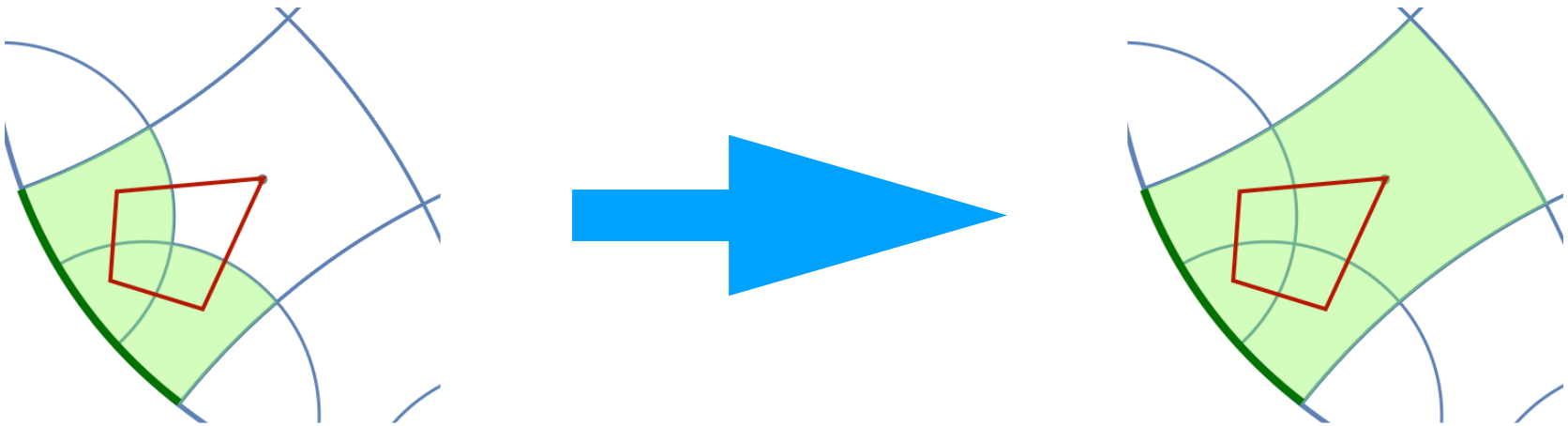




$$S_A = \frac{\text{Area of } \gamma_A}{4G_N} (+\text{bulk corrections})$$

1. Rindler reconstruction
2. Mutual information obeys RT formula
3. Black hole behavior (entropy and microstate recovery)
4. Holography (isometry between bulk and boundary) at finite T

AdS-Rindler Reconstruction



$$\mathcal{O}_p = \prod_{i=1}^{4} S_i^z$$

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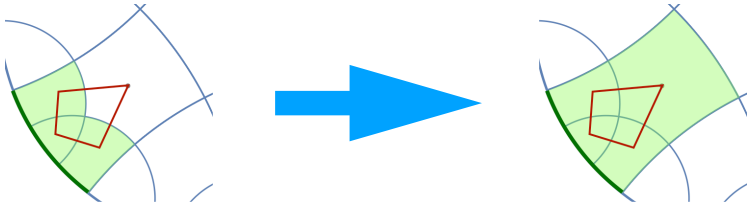

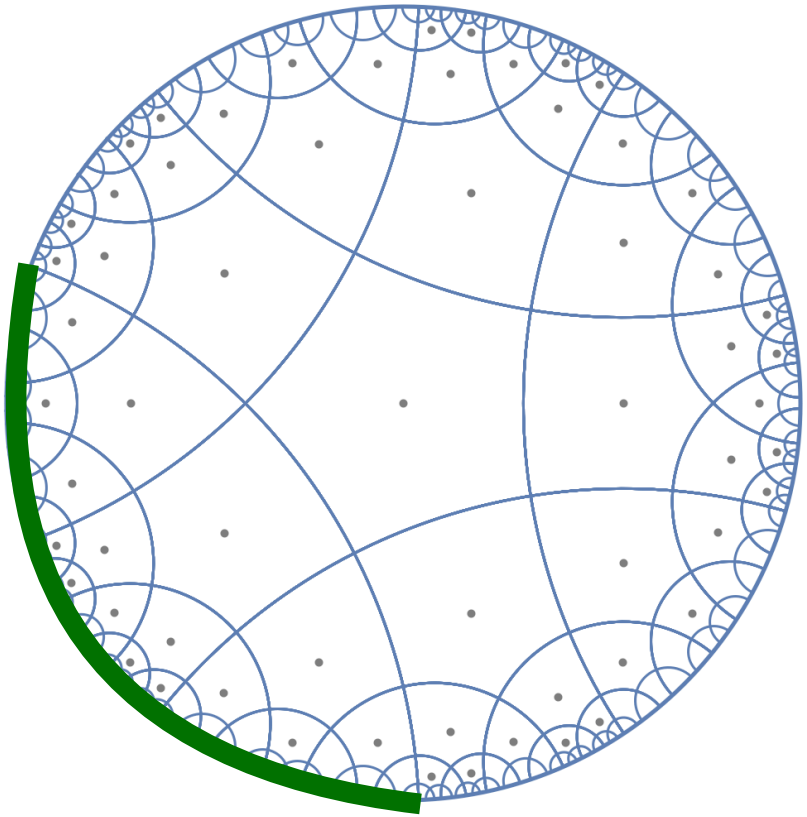

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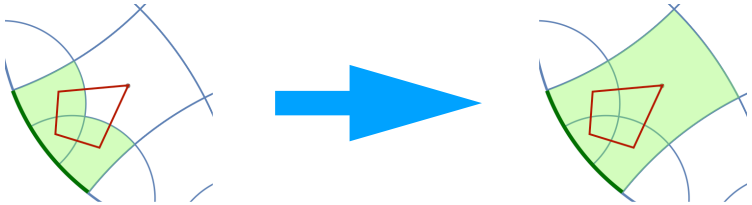
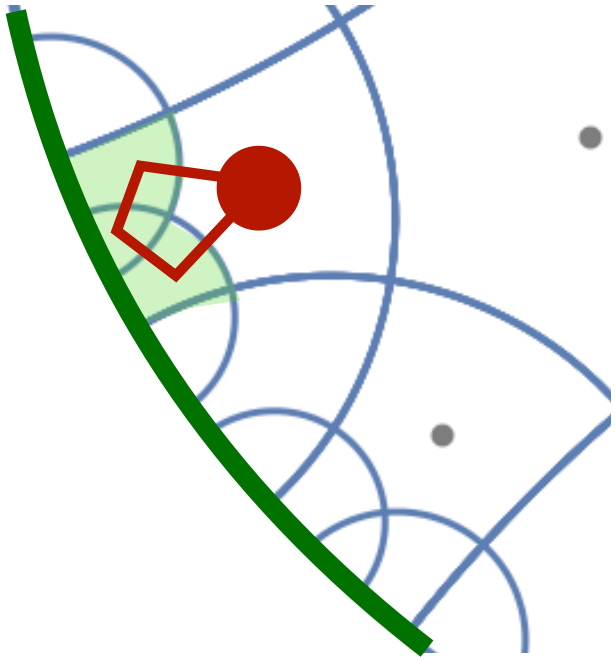
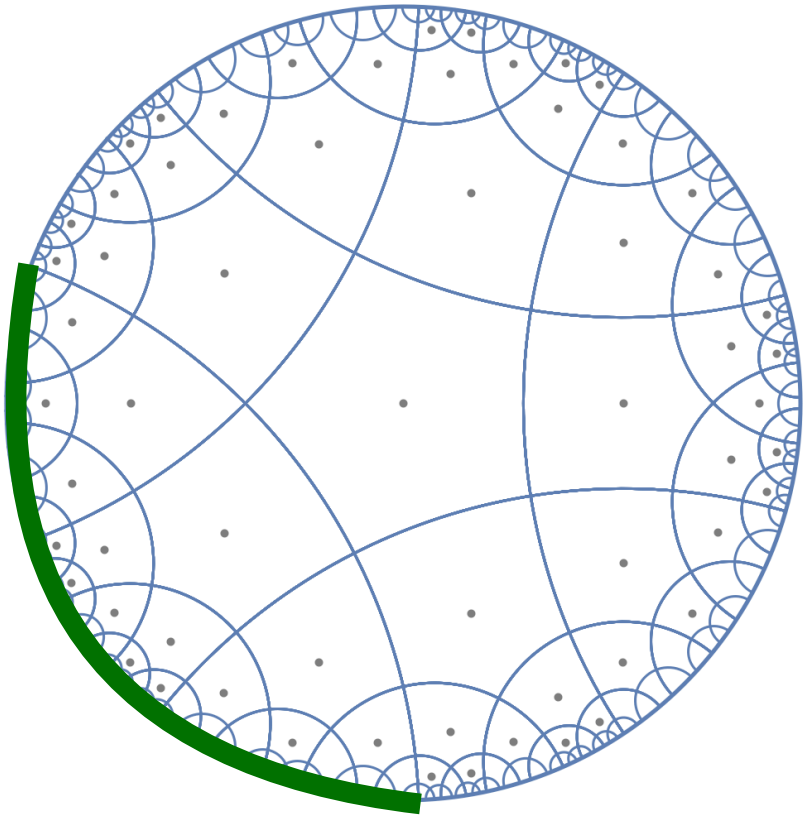
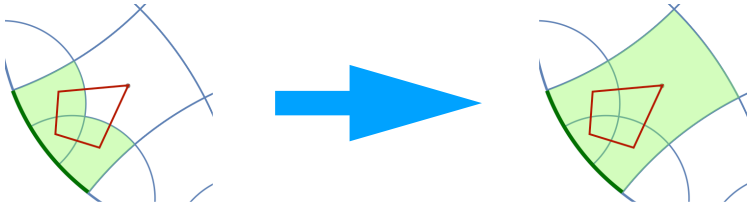
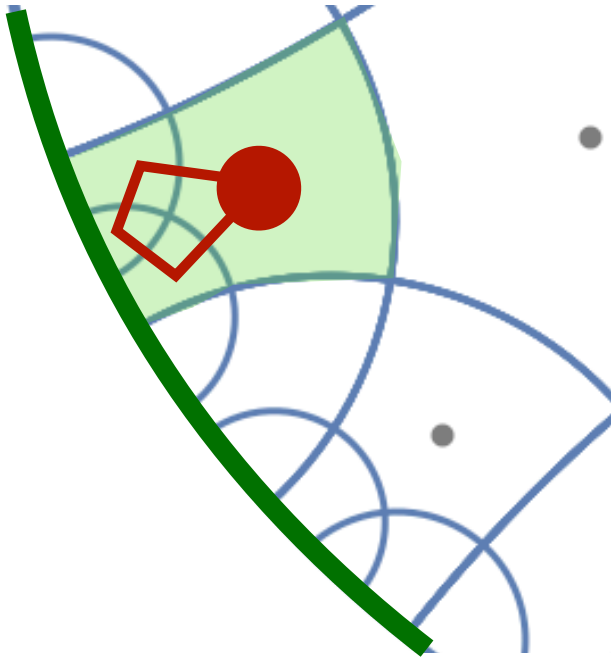


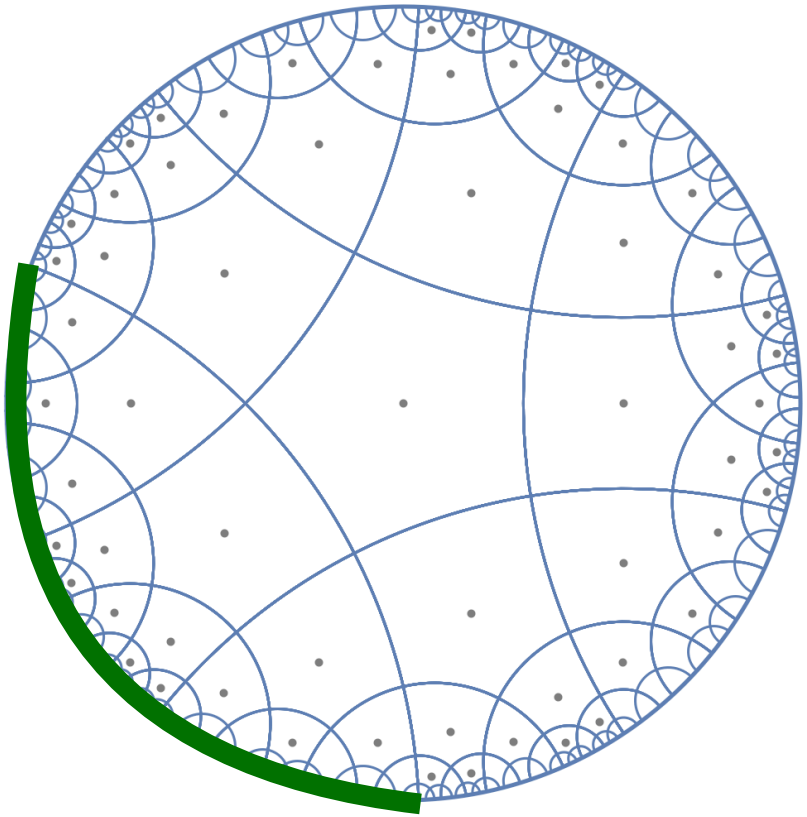




AdS-Rindler Reconstruction







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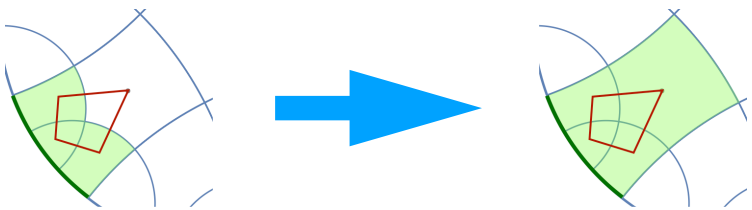
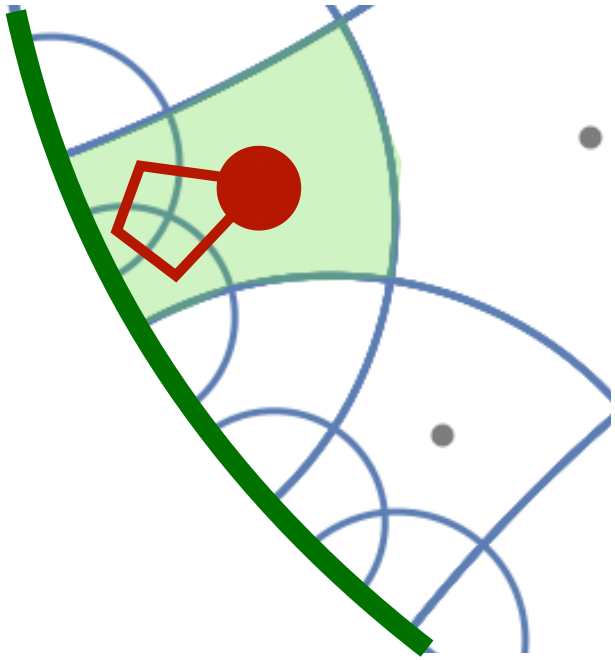
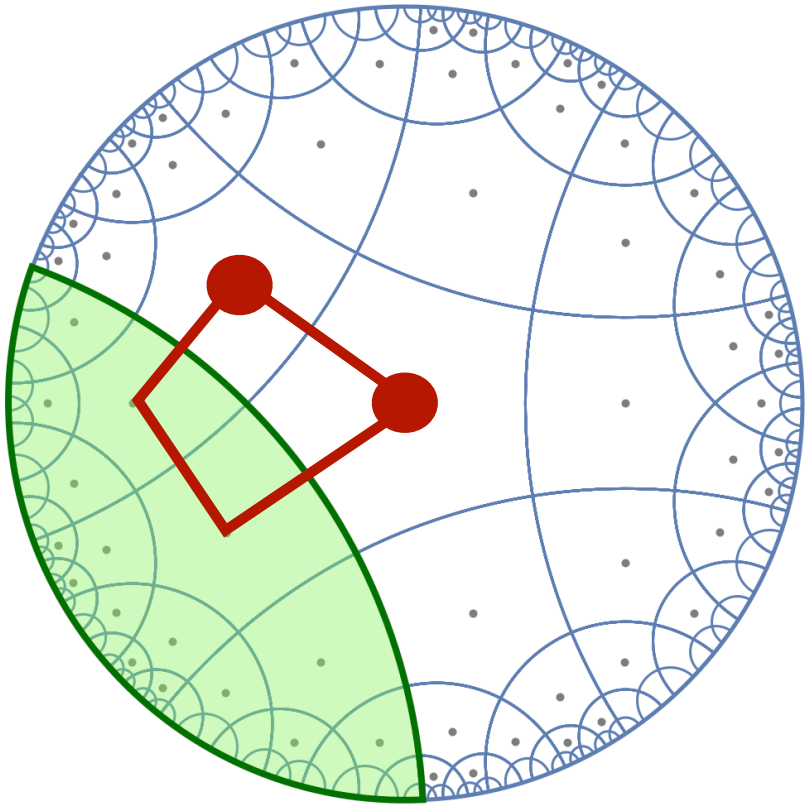




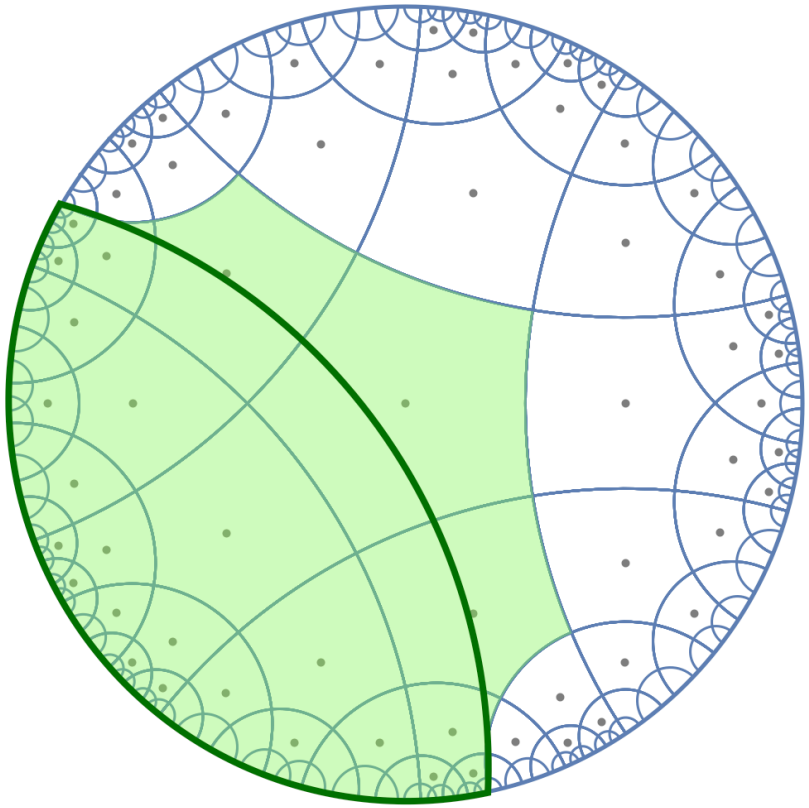


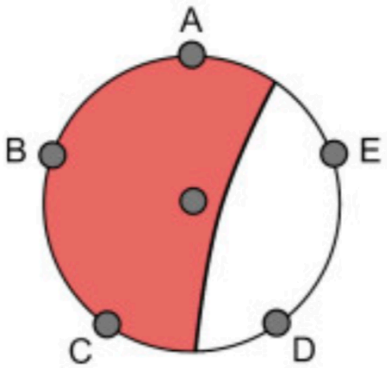
AdS-Rindler Reconstruction

For generic boundary region:

all bulk pentagons having non-zero overlap with geodesic wedge

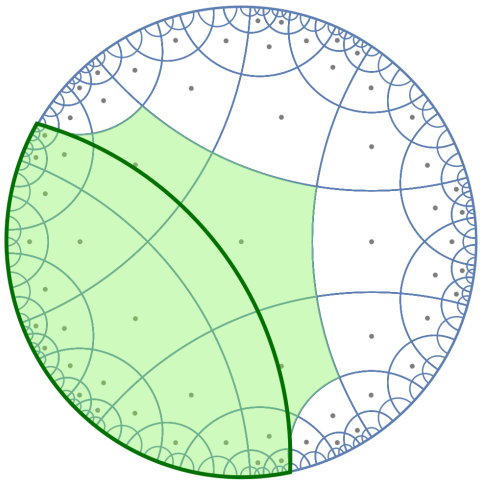
or **minimal convex wedge**





Subsystem symmetry is the key

FM Ising Model	one boundary point fixes the entire bulk	log(2)
Fracton Model	a boundary section fixes and fixes only the geodesic wedge.	~boundary area
Classical Lattice Gauge Model (with local d.o.f.)	cannot fix bulk sites at all, due to local degrees of freedom.	~bulk volume



Mutual Information

Mutual Information

The classical analog of **entanglement entropy** is **mutual information:**

$$I_{\mathrm{cl}}(A;B) = S_{\mathrm{s}}(A) + S_{\mathrm{s}}(B) - S_{\mathrm{s}}(A,B)$$

A & B: boundary sections.
S_S: Shannon entropy
Its quantum version:

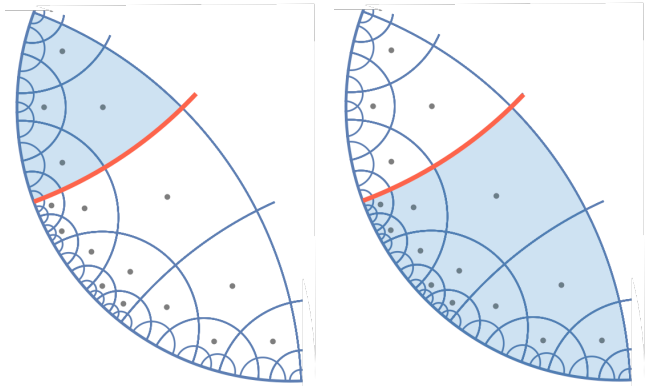
$$I_{\mathrm{qu}}(A;B) = S_{\mathrm{v}}(A) + S_{\mathrm{v}}(B) - S_{\mathrm{v}}(A \cup B)$$

is twice the entanglement entropy, for
S_v: von Neumann entropy, $B=A^c$

Mutual Information & RT formula

$$I_{\mathrm{cl}}(A;B) = S_{\mathrm{s}}(A) + S_{\mathrm{s}}(B) - S_{\mathrm{s}}(A,B)$$

Every geodesic ending in A:
multiply the degeneracy by 2



$S_S(A)$ ~ number of geodesics ending in A

Mutual Information & RT formula

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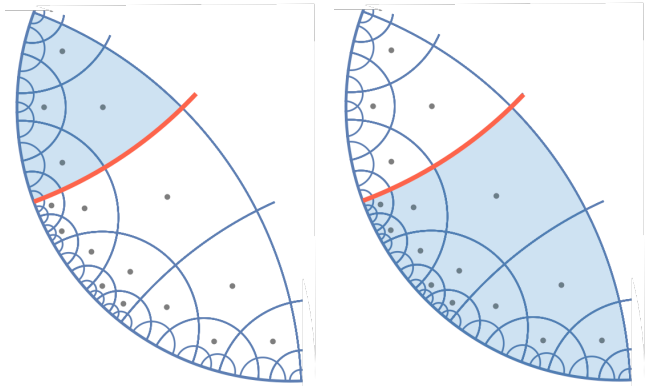
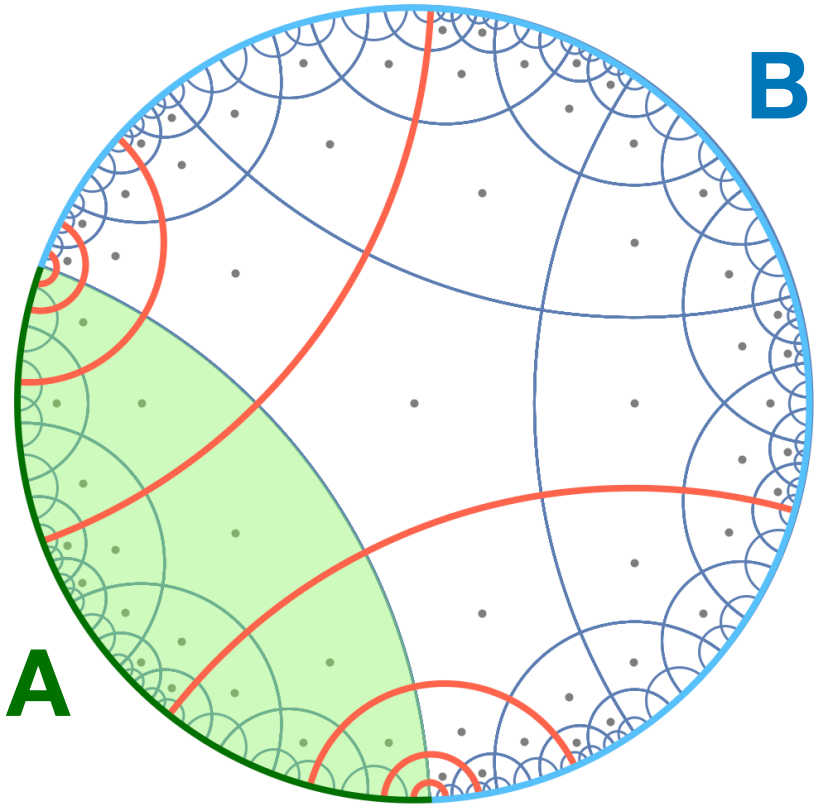


$S_S(A)$ ~ number of geodesics ending in A

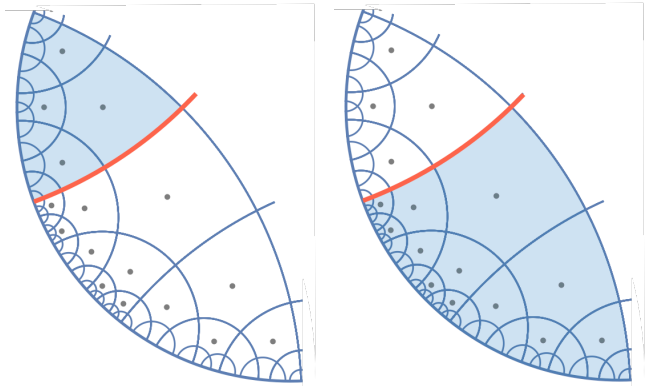


$I_{cl}(A)$ ~ number of geodesics crossing A and B

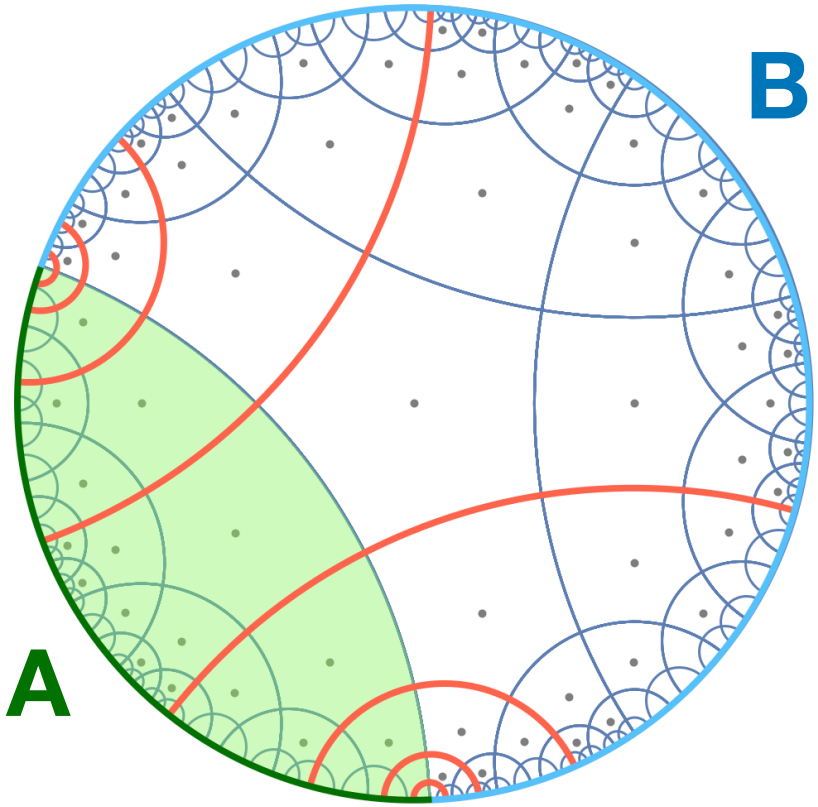
Mutual Information & RT formula

$$I_{\rm cl}(A;B) = S_{\rm s}(A) + S_{\rm s}(B) - S_{\rm s}(A,B)$$

Every geodesic ending in A: multiply the degeneracy by 2



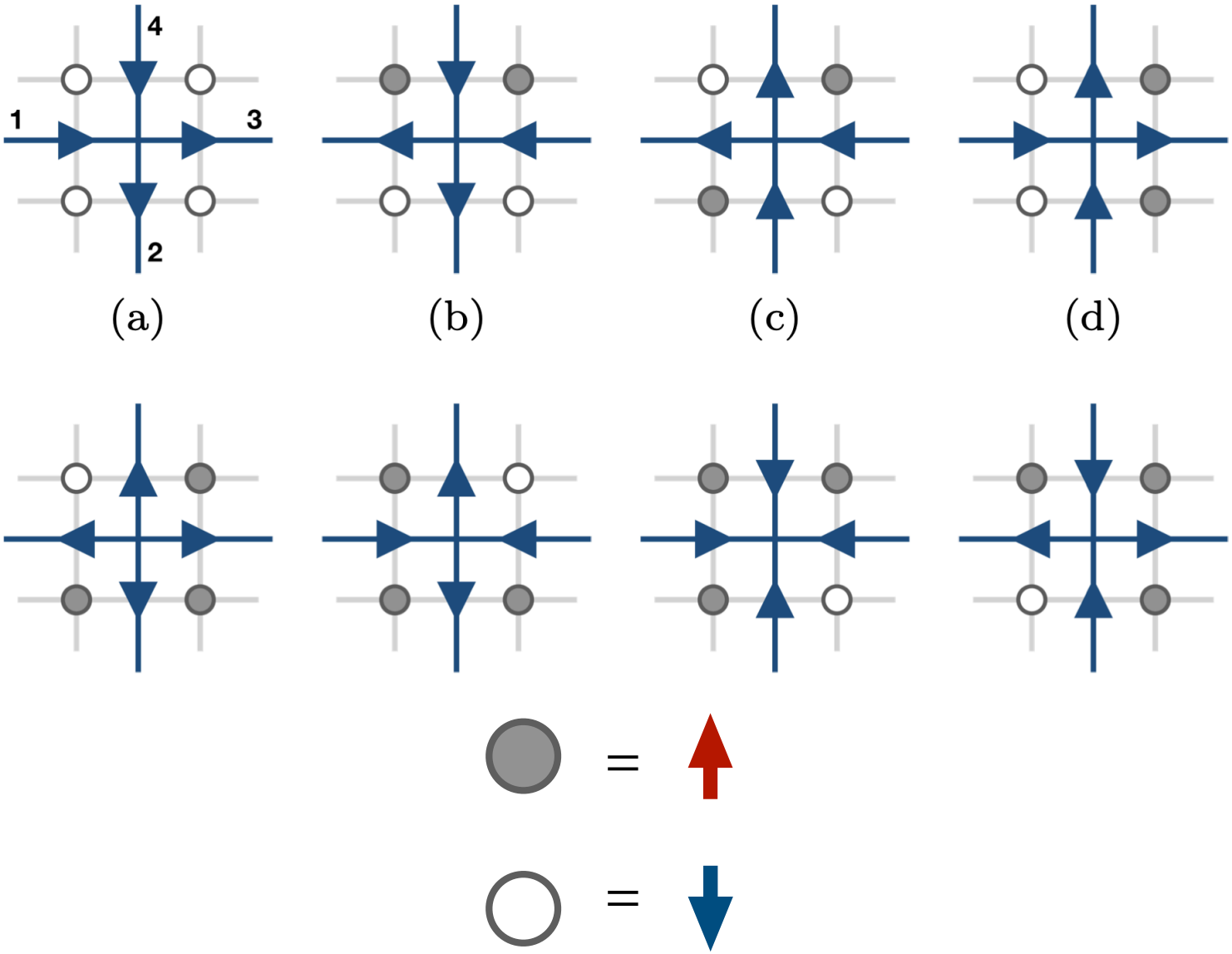
$S_S(A)$ ~ number of geodesics ending in A



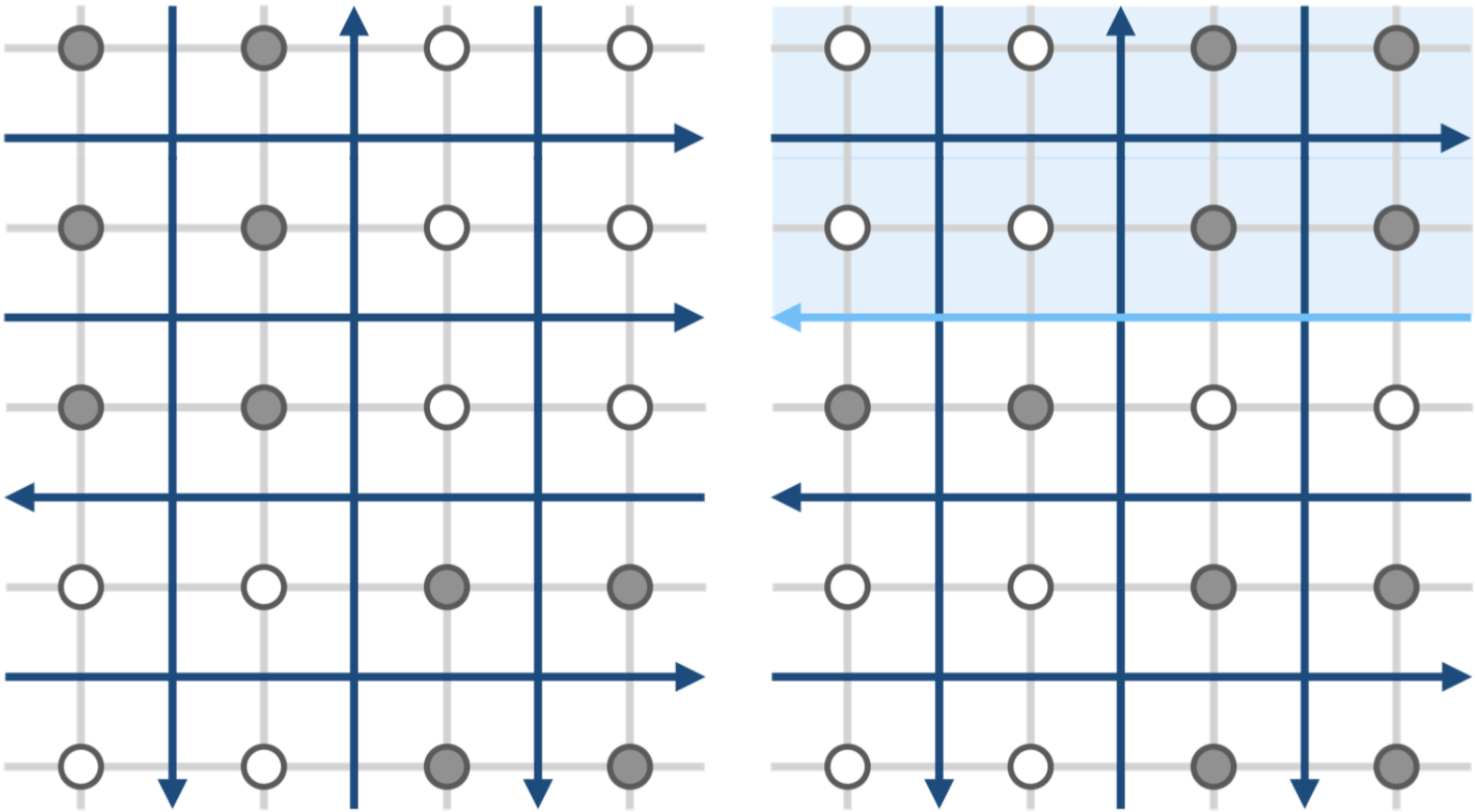
$I_{cl}(A)$ ~ number of geodesics crossing A and B
~ area (length) of γ

Clearer picture in a dual language

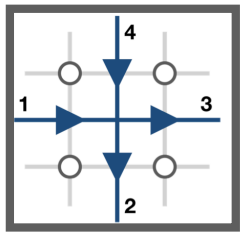
Dual Eight-Vertex Model



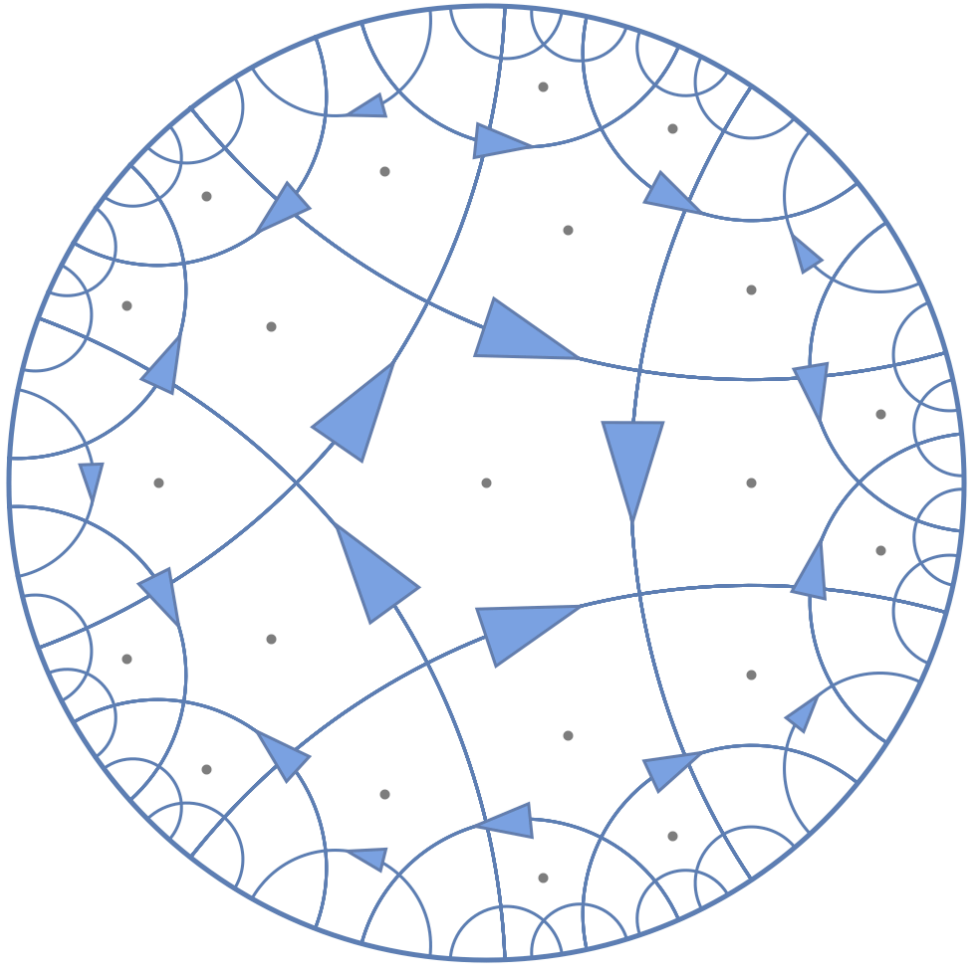
Dual Eight-Vertex Model



$$\mathcal{H}_{\text{spin}} = -\sum_p \mathcal{O}_p \qquad \mathcal{H}_{\text{EV}} = -\frac{1}{2}\sum_v (\sigma_1\sigma_3 + \sigma_2\sigma_4)$$



Dual Eight-Vertex Model



Dual eight-vertex model = classical bit threads

Outline

I. Brief introduction of holography

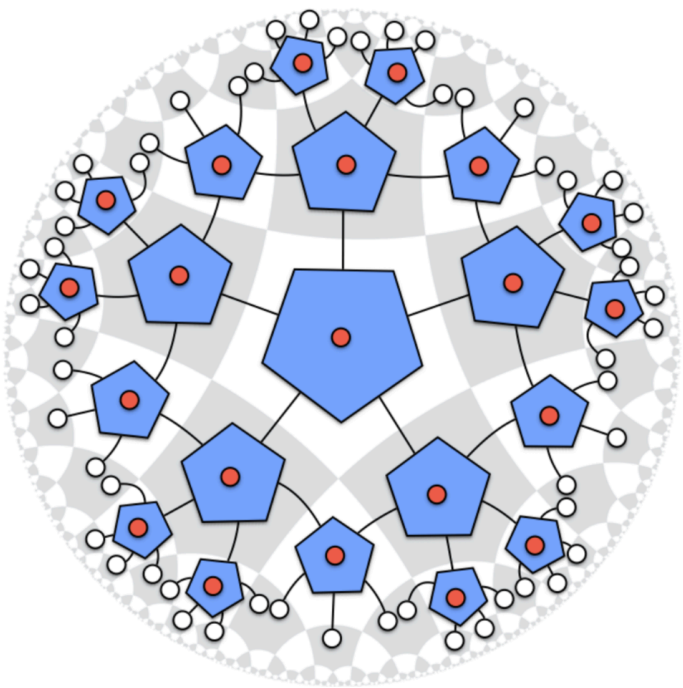
II. The classical fracton model on flat and hyperbolic lattices

III. Holographic properties of the hyperbolic fracton model

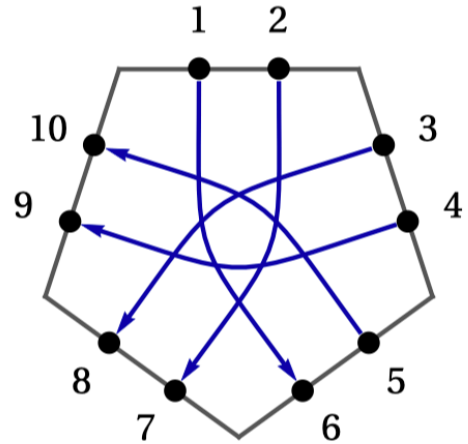
IV. Unifying different holographic toy models by Lifshitz gravity/symmetric rank-2 U(1) gauge theory

Emerging Unified Picture

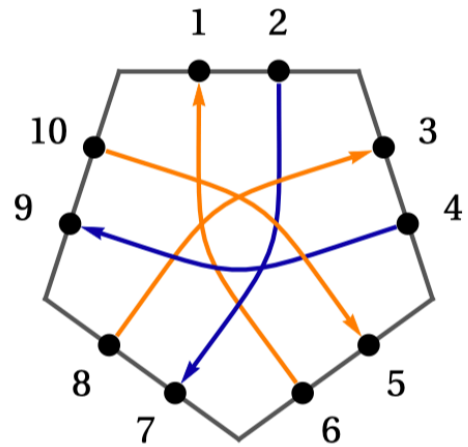
[Jahn , Gluza, Pastawski, Eisert, …]



$|\bar{0}\rangle_5 =$



$|\bar{1}\rangle_5 =$

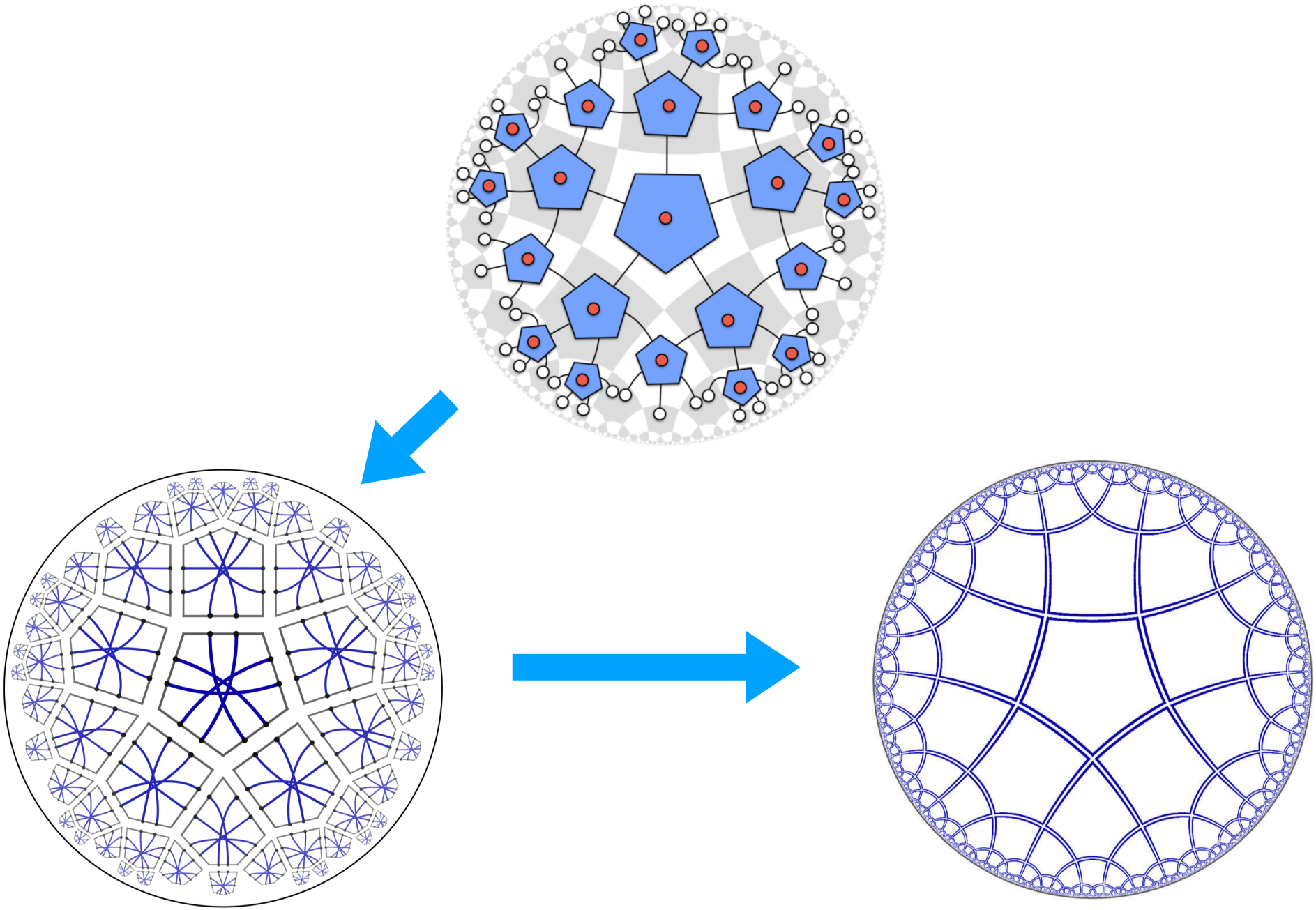


$$
\begin{aligned}
S_1 &= \sigma^x \otimes \sigma^z \otimes \sigma^z \otimes \sigma^x \otimes 1_2 = \mathrm{i}\gamma_7\gamma_2 \\
S_2 &= 1_2 \otimes \sigma^x \otimes \sigma^z \otimes \sigma^z \otimes \sigma^x = \mathrm{i}\gamma_9\gamma_4 \\
S_3 &= \sigma^x \otimes 1_2 \otimes \sigma^x \otimes \sigma^z \otimes \sigma^z = \mathrm{i}\mathcal{P}_{\mathrm{tot}}\gamma_6\gamma_1 \\
S_4 &= \sigma^z \otimes \sigma^x \otimes 1_2 \otimes \sigma^x \otimes \sigma^z = \mathrm{i}\mathcal{P}_{\mathrm{tot}}\gamma_8\gamma_3 \\
S_5 &= \sigma^z \otimes \sigma^z \otimes \sigma^x \otimes 1_2 \otimes \sigma^x = \mathrm{i}\mathcal{P}_{\mathrm{tot}}\gamma_{10}\gamma_5
\end{aligned}
$$

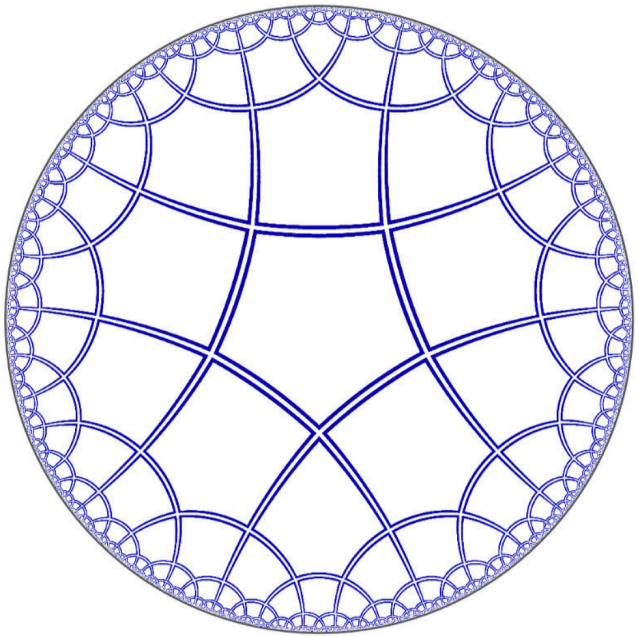
$$
\Gamma^{\psi}_{j,k} = \frac{\mathrm{i}}{2} \langle \psi | [\gamma_j, \gamma_k] | \psi \rangle
$$

$$
\Gamma^{\psi}_{j,k} = \begin{cases} -1 & \text{for an arrow } j \to k \\ 1 & \text{for an arrow } k \to j \\ 0 & \text{if no arrow connects } j \text{ and } k \end{cases}
$$

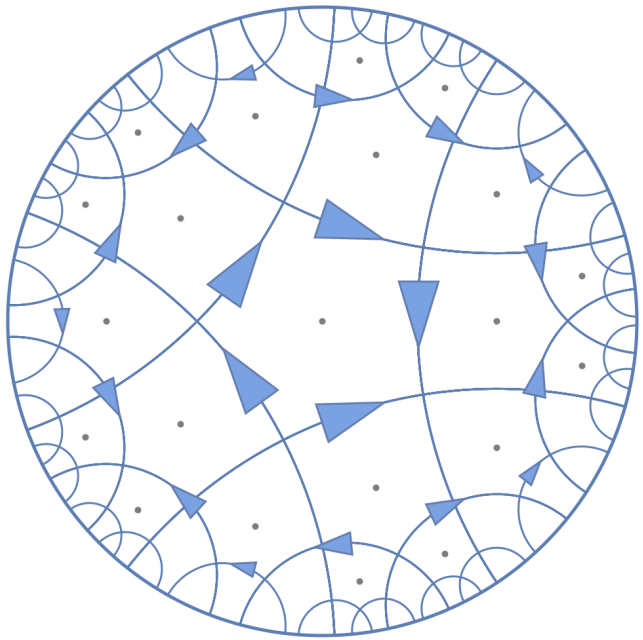
Emerging Unified Picture



Emerging Unified Picture



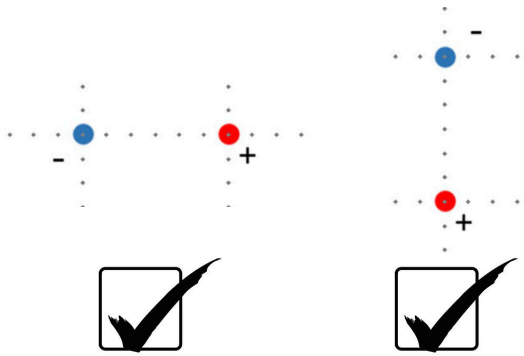
Tensor network



fracton model and
bit threads

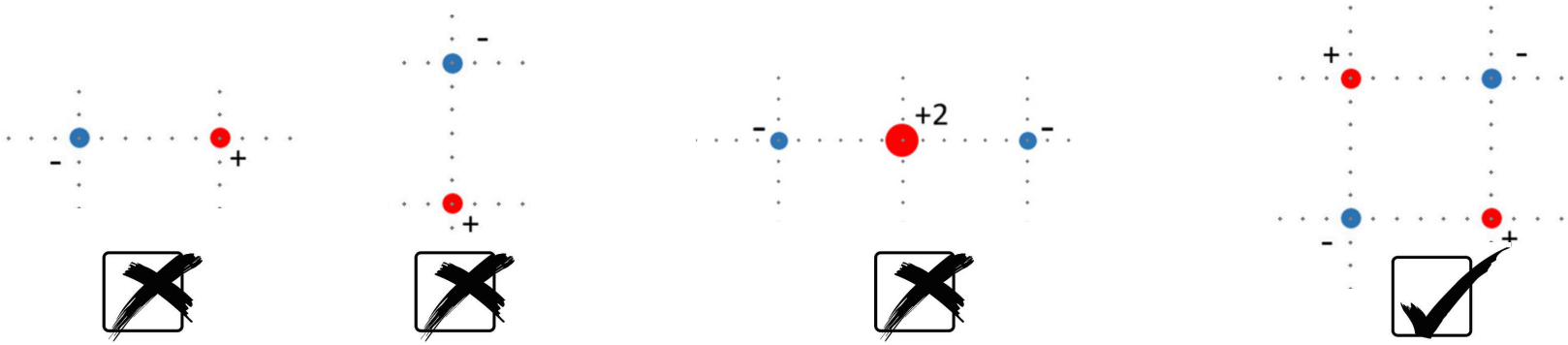
RANK-2 U(1): THE UNIFYING THEORY

Conventional U(1) $\int \rho = \text{const.},$



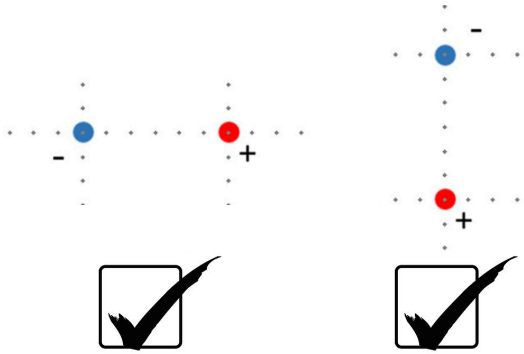
Rank-2 U(1) $\partial_i \partial_j E^{ij} = \rho \quad E^i{}_i = 0$ → $\int \rho = \text{const.}, \quad \int \vec{x}\rho = \text{const.}, \quad \int x^2 \rho = \text{const.}$

$$A_{ij} \to \partial_i \partial_j \lambda + \delta_{ij} \gamma$$



Rank-2 U(1): the unifying theory

Conventional U(1) $\int \rho = \text{const.},$



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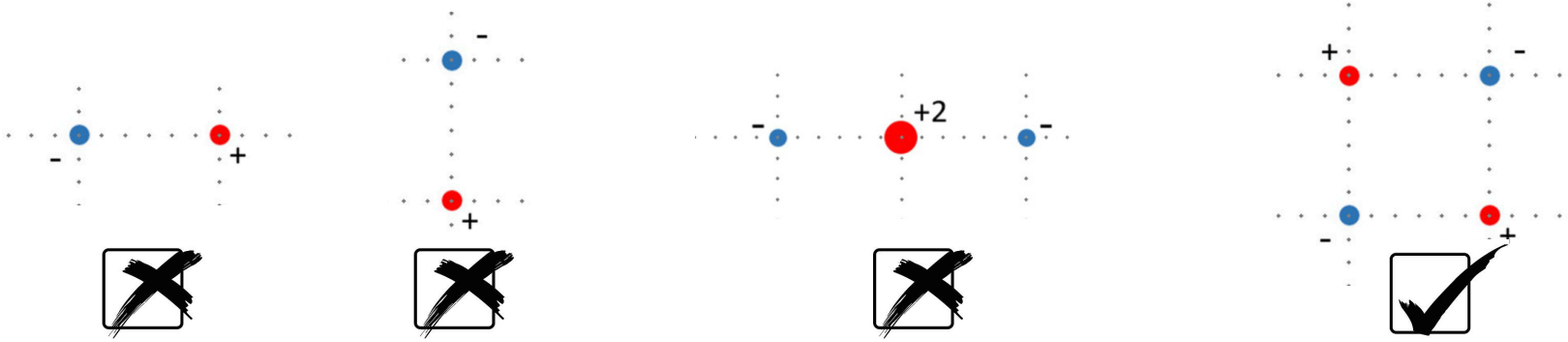

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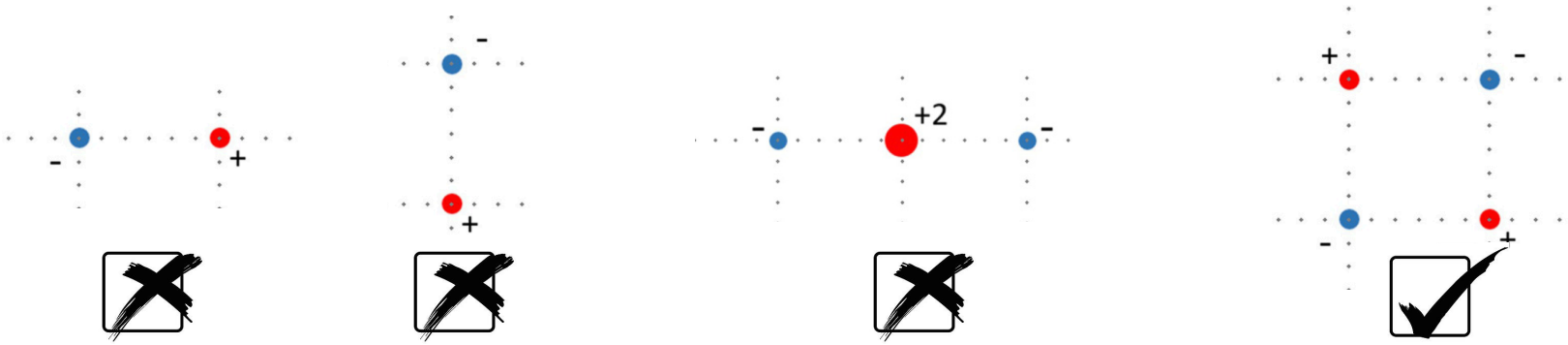
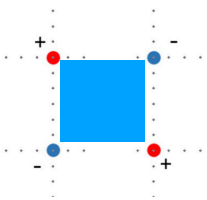
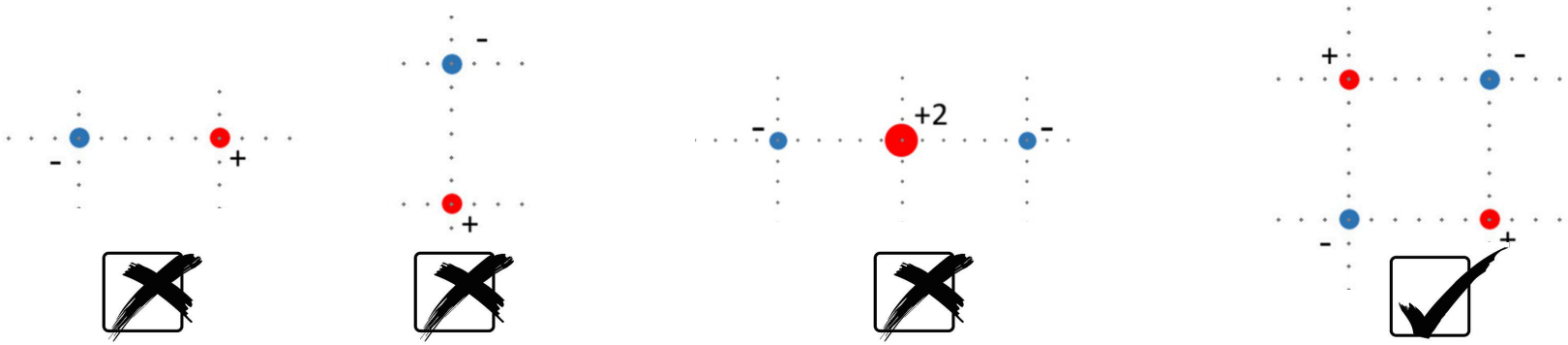


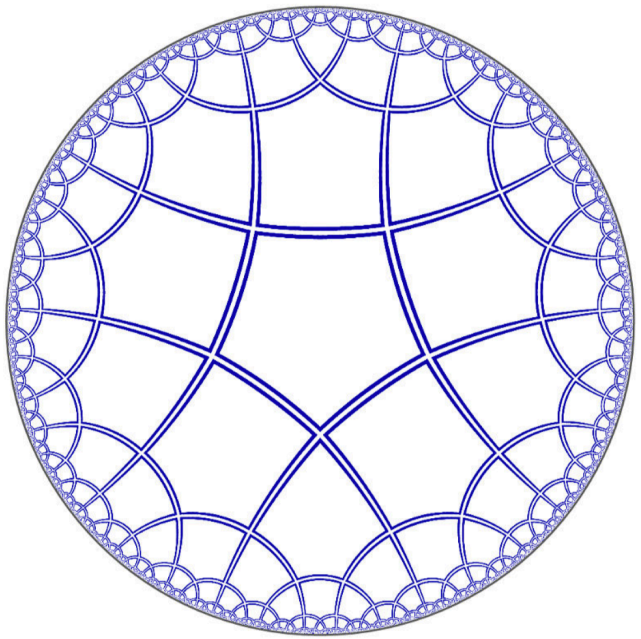


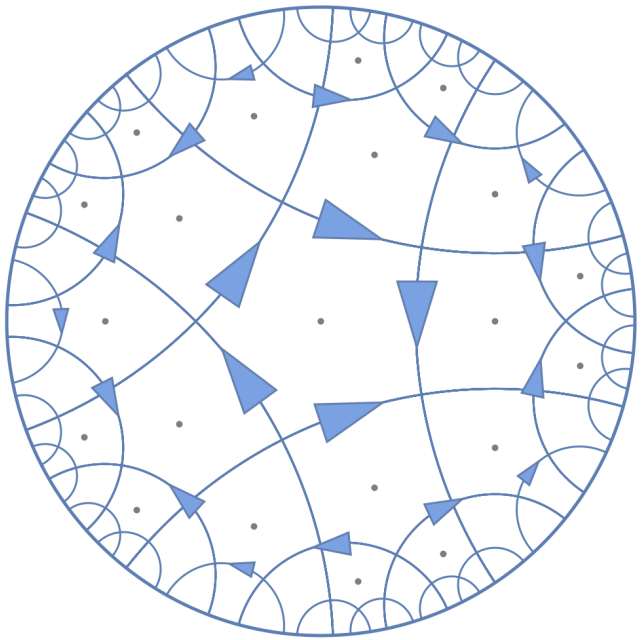
RANK-2 U(1): THE UNIFYING THEORY

Rank-2 U(1) $\partial_i \partial_j E^{ij} = \rho \quad E^i{}_i = 0$ → $\int \rho = \text{const.}, \quad \int \vec{x}\rho = \text{const.}, \quad \int x^2 \rho = \text{const.}$

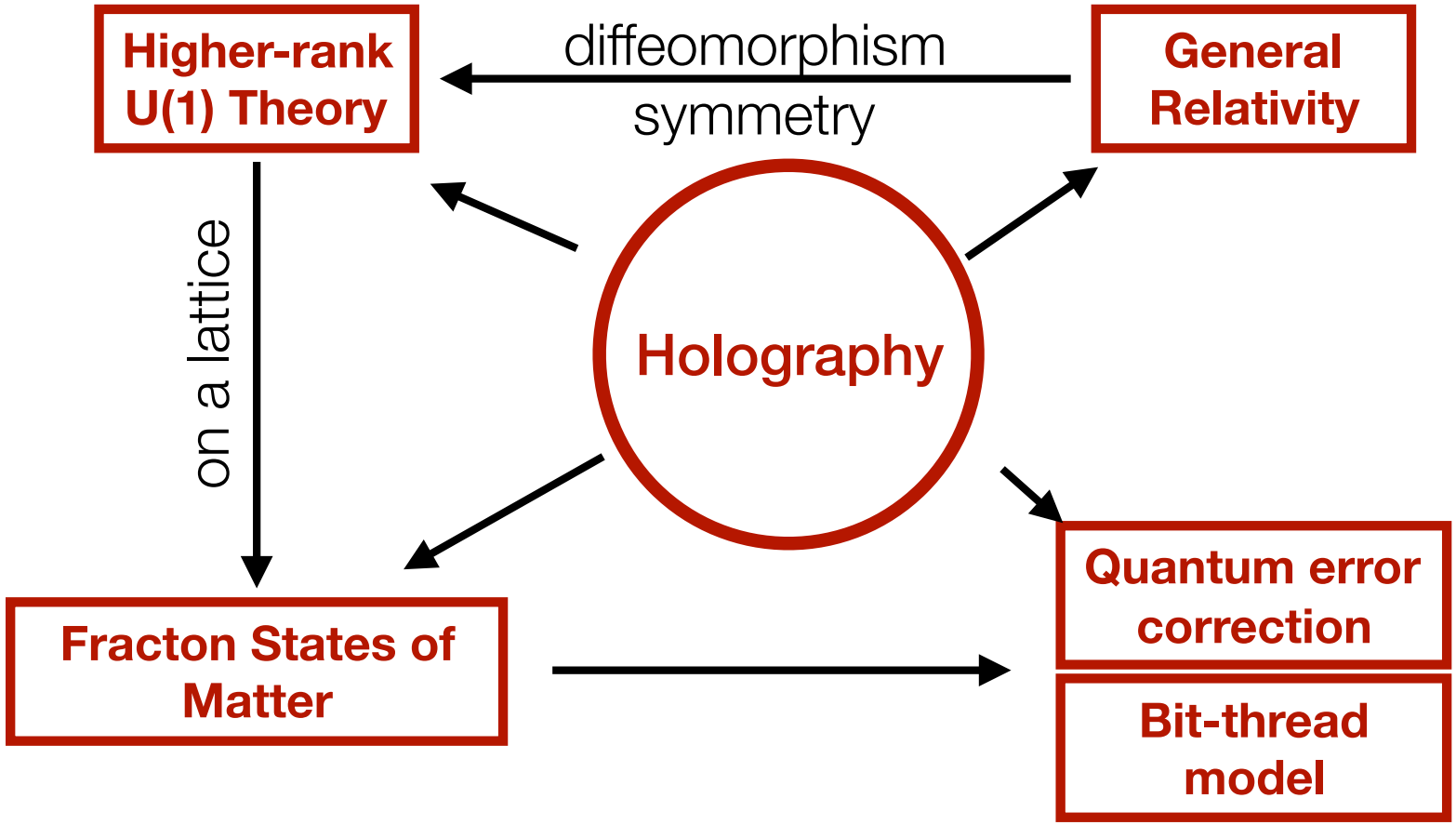


Rank-2 U(1): the unifying theory

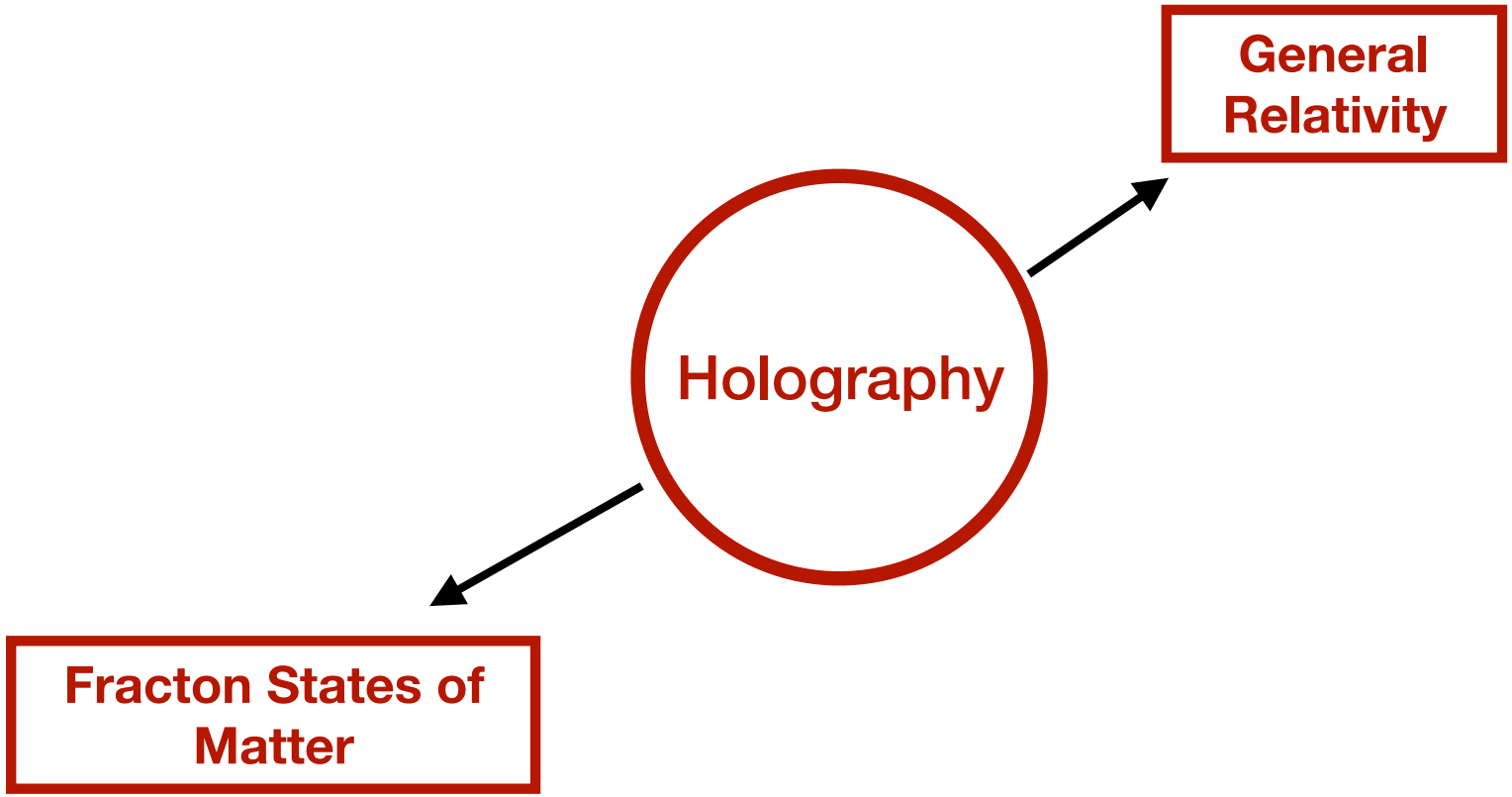




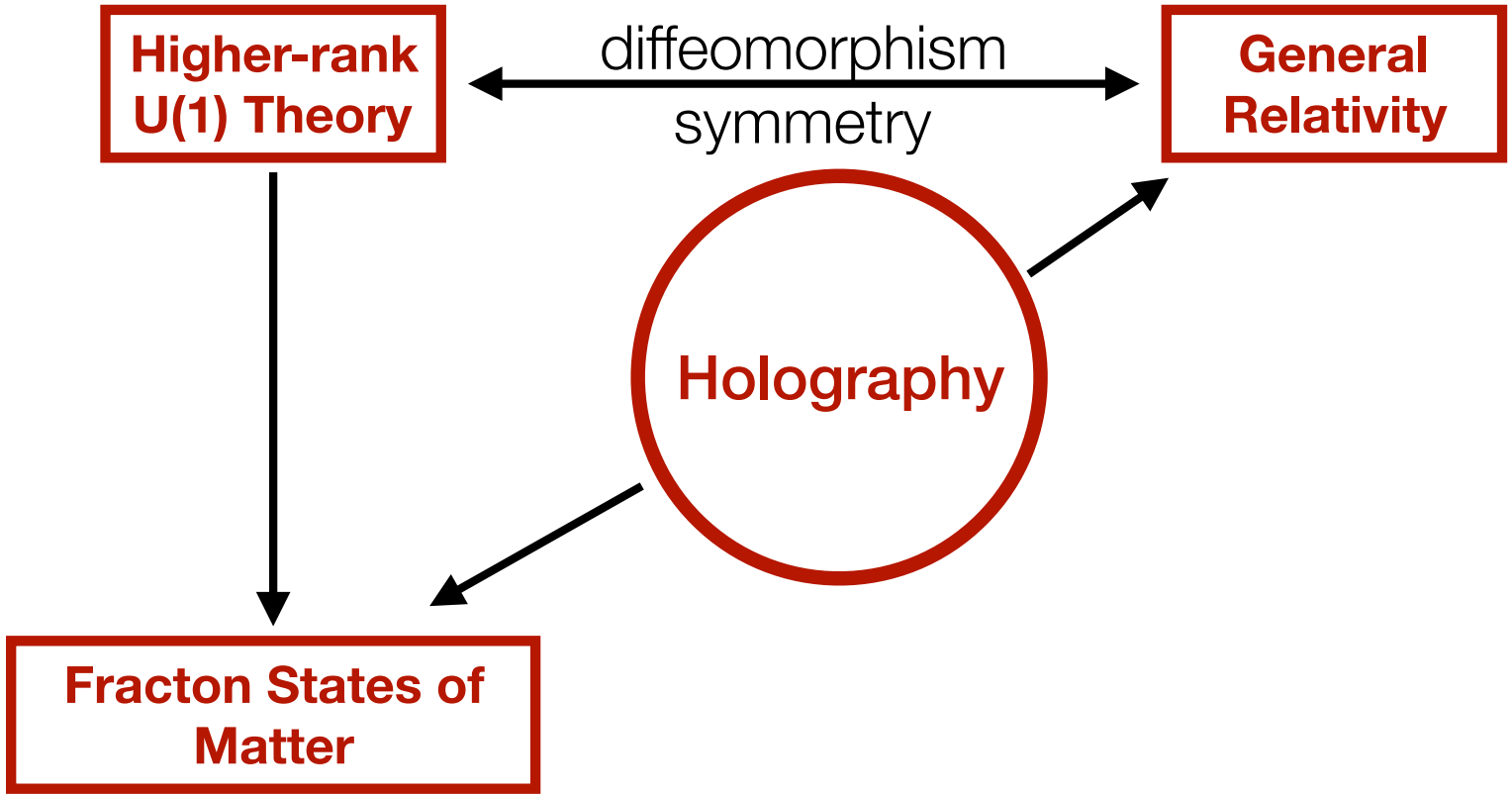
Speculated Web of Connections



Speculated Web of Connections



Speculated Web of Connections



Thank you!