FRACTON STATES OF MATTER AS TOY MODELS OF HOLOGRAPHY

+ work in progress with A.Prem (Princeton) and A.Thomas (CU Boulder)

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Is there a many-body system that mimics the exotic properties of gravity —
Is there a many-body system that mimics the exotic properties of gravity — in the field theory aspect (gravitons, scattering etc)?
The Question

Is there a many-body system that mimics the exotic properties of gravity —

- in the field theory aspect (gravitons, scattering etc)?

- in the informational aspect (entanglement structure)?
I. Brief introduction of holography

II. The classical fracton model on flat and hyperbolic lattices

III. Holographic properties of the hyperbolic fracton model

IV. Unifying different holographic toy models by Lifshitz gravity/symmetric rank-2 U(1) gauge theory
Entanglement between $A$ & $A_c$

$$S_A = \frac{\text{Area of } \gamma_A}{4G_N} (+\text{bulk corrections})$$

[Ryu, Takayanagi]
AdS-Rindler reconstruction:
Given information on boundary region $A$, bulk information in the \textit{geodesic wedge} $W(A)$ can be reconstructed; bulk information beyond the \textit{geodesic wedge} $W(A)$ cannot.
RINDLER RECONSTRUCTION

Non-trivial encoding:
Center bulk op. is in the geodesic wedge of AB, BC, CA but not the wedge of A, B, or C individually
Perfect tensor networks and random tensor networks

[HaPPY (Harlow, Preskill, Pastawski, Yoshida), Qi, ...]
Known Toy Models

Bit-thread constructions

[Freedman, Headrick, Wen, ...]
A classical many-body toy model (fracton states) mimics the exotic properties of gravity —

◼️ in the field theory aspect (gravitons)?

☑️ in the informational aspect (entanglement structure)?
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[Chen, Slagle, Shirley, Xu, Fu, Vijay, Balents, Pretko, Haah, Halasz, Hsieh, Ma, Kim...]

OUTLINE
A BIT OF HISTORY

1. Gauge theory with symmetric rank-2 U(1) electric fields [Xu, …]
2. R2-U1 found to be some versions of Lifshitz gravity [Xu, Horava …]
3. Fracton topological order was discovered [Fu, Vijay, Haah,…]
4. R2-U1 + Higgs = gapped Fracton topological order / other fracton states, featuring subsystem symmetries [Chen, Slagle, Pretko, Ma, …]
Fracton Model on a Flat Lattice

Sites on the center of square plaquette,
Fracton Model on a Flat Lattice

Ground state: $O_p = 1$ for all four-spin clusters

Ground state degeneracy: $\sim 2^{L_x + L_y}$
Ground state: $O_p=1$ for all four-spin clusters

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Ground state: $O_p=1$ for all four-spin clusters

Ground state degeneracy: $\sim 2^{L_x+L_y}$
**Fracton Model on a Flat Lattice**

First excited state: a fracton
\( O_p = -1 \) for one plaquette
\( O_p = 1 \) for all other plaquettes

Cannot be created by local operation (without extra energy cost)

Cannot be moved by local operation (without extra energy cost)
Bound states of fractons:

2-fracton bound state: can move in 1D

4-fracton bound state: can move more freely
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2-fracton bound state:
can move in 1D

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can more freely
Fracton Model on a Flat Lattice

Sub-extensive ground state degeneracy (subsystem symmetry)

Fracton excitations can only move in a sub-manifold of the system

Bound states of fractons can move in higher dimensions
The Model

Sites on the center of pentagon form a square lattice

\[ \mathcal{O}_p = \prod_{i=1}^{4} S_i^z \]

\[ H = - \sum_p \mathcal{O}_p \]

Geodesics (straight lines) are edges of pentagons
Lattice of different size: characterized by number of geodesics $N_g$
Ground State Degeneracy

Ground state:
Flip an entire geodesic wedge.
Degeneracy: $2^{N_g + 1}$
1-fracton state:
Flip a region covered by intersecting geodesics
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1. Rindler reconstruction

2. Mutual information obeys RT formula

3. Black hole behavior (entropy and microstate recovery)

4. Holography (isometry between bulk and boundary) at finite $T$
**AdS–Rindler Reconstruction**

\[
O_p = \prod_{i=1}^{4} S^z_i
\]
AdS–Rindler Reconstruction
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AdS–Rindler Reconstruction
AdS–Rindler Reconstruction
For generic boundary region:

all bulk pentagons having non-zero overlap with geodesic wedge

or **minimal convex wedge**
<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>FM Ising Model</td>
<td>one boundary point fixes the entire bulk</td>
<td>$\log(2)$</td>
</tr>
<tr>
<td>Fracton Model</td>
<td>a boundary section fixes and fixes only the geodesic wedge.</td>
<td>$\sim$ boundary area</td>
</tr>
<tr>
<td>Classical Lattice Gauge Model (with local d.o.f.)</td>
<td>cannot fix bulk sites at all, due to local degrees of freedom.</td>
<td>$\sim$ bulk volume</td>
</tr>
</tbody>
</table>

**Subsystem Symmetry is the Key**

![Diagram](image)
MUTUAL INFORMATION
The classical analog of entanglement entropy is mutual information:

\[ I_{cl}(A; B) = S_s(A) + S_s(B) - S_s(A, B) \]

\( A \& B \): boundary sections.
\( S_s \): Shannon entropy

Its quantum version:

\[ I_{qu}(A; B) = S_v(A) + S_v(B) - S_v(A \cup B) \]

is twice the entanglement entropy, for
\( S_v \): von Neumann entropy, \( B = A^c \)
Mutual Information & RT Formula

\[ I_{\text{cl}}(A; B) = S_s(A) + S_s(B) - S_s(A, B) \]

Every geodesic ending in A: multiply the degeneracy by 2

\[ S_s(A) \sim \text{number of geodesics ending in A} \]
Every geodesic ending in A: multiply the degeneracy by 2

\[ I_{cl}(A; B) = S_s(A) + S_s(B) - S_s(A, B) \]

\( S_s(A) \sim \text{number of geodesics ending in } A \)

\( I_{cl}(A) \sim \text{number of geodesics crossing } A \text{ and } B \)
Every geodesic ending in $A$: multiply the degeneracy by 2

$I_{cl}(A; B) = S_s(A) + S_s(B) - S_s(A, B)$

Every geodesic ending in $A$: multiply the degeneracy by 2

$I_{cl}(A) \sim$ number of geodesics crossing $A$ and $B$

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$I_{cl}(A) \sim$ area (length) of $\gamma$
CLEARER PICTURE IN A DUAL LANGUAGE
DUAL EIGHT-VERTEX MODEL

(a)  

(b)  

(c)  

(d)  

\[ \begin{array}{c}
\text{\textbullet} = \uparrow \\
\text{\textcircled{\(\textbullet\)}} = \downarrow
\end{array} \]
**Dual Eight-Vertex Model**

\[
\mathcal{H}_{\text{spin}} = - \sum_p O_p \quad \mathcal{H}_{\text{EV}} = -\frac{1}{2} \sum_v (\sigma_1 \sigma_3 + \sigma_2 \sigma_4)
\]
Dual eight-vertex model = classical bit threads
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EMERGING UNIFIED PICTURE

[Jahn, Gluza, Pastawski, Eisert, …]

\[ |0\rangle_5 = \]

\[ |1\rangle_5 = \]

\[
S_1 = \sigma^x \sigma^z \sigma^z \sigma^x \sigma^x 1_2 = i \gamma_7 \gamma_2 \\
S_2 = 1_2 \sigma^x \sigma^z \sigma^z \sigma^x \sigma^x = i \gamma_9 \gamma_4 \\
S_3 = \sigma^x 1_2 \sigma^x \sigma^z \sigma^z \sigma^x = i \mathcal{P}_{\text{tot}} \gamma_6 \gamma_1 \\
S_4 = \sigma^z \sigma^x \sigma^x 1_2 \sigma^x \sigma^z = i \mathcal{P}_{\text{tot}} \gamma_8 \gamma_3 \\
S_5 = \sigma^z \sigma^z \sigma^x \sigma^x 1_2 \sigma^x = i \mathcal{P}_{\text{tot}} \gamma_{10} \gamma_5
\]

\[
\Gamma^\psi_{j,k} = \left\{ \begin{array}{cl}
-1 & \text{for an arrow } j \rightarrow k \\
1 & \text{for an arrow } k \rightarrow j \\
0 & \text{if no arrow connects } j \text{ and } k
\end{array} \right.
\]
EMERGING UNIFIED PICTURE
EMERGING UNIFIED PICTURE

Tensor network

fracton model and bit threads
**Rank-2 U(1): The Unifying Theory**

Conventional $U(1)$

$$\int \rho = \text{const.,}$$

Rank-2 $U(1)$

$$\partial_i \partial_j E^{ij} = \rho \quad E^i_i = 0$$

$$A_{ij} \rightarrow \partial_i \partial_j \lambda + \delta_{ij} \gamma$$

$$\int \rho = \text{const.,} \quad \int \bar{x} \rho = \text{const.,} \quad \int x^2 \rho = \text{const.}$$
**Rank-2 U(1): The Unifying Theory**

Conventional U(1)

\[ \int \rho = \text{const.}, \]

\[ A_{ij} \rightarrow \partial_i \partial_j \lambda + \delta_{ij} \gamma \]

Rank-2 U(1)

\[ \partial_i \partial_j E^{ij} = \rho \quad E^i_i = 0 \]

\[ \int \rho = \text{const.}, \quad \int \tilde{x} \rho = \text{const.}, \quad \int x^2 \rho = \text{const.} \]
Rank-2 U(1): The Unifying Theory

Rank-2 $U(1)$: $\partial_i \partial_j E^{ij} = \rho$, $E^i_i = 0$ \( \Rightarrow \int \rho = \text{const.}, \int x^i \rho = \text{const.}, \int x^2 \rho = \text{const.} \)
**Rank-2 U(1): The Unifying Theory**

\[
\partial_i \partial_j E^{ij} = \rho \quad E_i^i = 0 \quad \Rightarrow \quad \int \rho = \text{const., } \int \vec{x} \rho = \text{const., } \int x^2 \rho = \text{const.}
\]
Rank-2 U(1): the unifying theory
Speculated Web of Connections

- Higher-rank U(1) Theory
- Fracton States of Matter
- General Relativity
- Quantum error correction
- Bit-thread model
Speculated Web of Connections

Fracton States of Matter

Holography

General Relativity
Speculated Web of Connections

Higher-rank U(1) Theory

Fracton States of Matter

Holography

General Relativity

diffeomorphism symmetry
THANK YOU!